Williamson's Woes

by

Neil Tennant*

Department of Philosophy
The Ohio State University
Columbus, Ohio 43210
email tennant.9@osu.edu

October 6, 2009

Abstract

This is a reply to Timothy Williamson's paper 'Tennant's Troubles'. It defends against Williamson's objections the anti-realist's knowability principle based on the author's 'local' restriction strategy involving Cartesian propositions, set out in *The Taming of the True*. Williamson's purported Fitchian *reductio*, involving the unknown number of books on his table, is analyzed in detail and shown to be fallacious. Williamson's attempt to cause problems for the anti-realist by means of a supposed rigid designator generates a contradiction with arithmetic right away, upon instantiating the obviously relevant theorem that every natural number is provably odd or provably even. The paper also explains and formulates a globally restricted knowability principle, which likewise blocks the attempted *reductio*.

*Thanks to Salvatore Florio, Nicholas Jones, Robert Kraut, Kevin Scharp, Joshua Smith and Tadeusz Szubka for helpful comments and discussions.

1 The anti-realist's Knowability Principle

The rule of inference

$$(\Diamond K)\ \frac{\varphi}{\Diamond K\varphi}$$

expresses the anti-realist's original, unrestricted Knowability Principle, to the effect that every truth is knowable. In Tennant (1997), Chapter 8, it was proposed, in light of Fitch's paradox, that this rule be restricted in its applications, to so-called *Cartesian* propositions. A proposition φ is Cartesian just in case $K\varphi \nvdash \bot$. So the restricted Knowability Principle is:

$$(\Diamond KC)\ \frac{\varphi}{\Diamond K\varphi}\ \text{where } K\varphi \nvdash \bot$$

The turnstile here is that of logico-mathematical deducibility.

2 Background to the present discussion

The background to the present discussion is the following sequence of subsequent writings: Timothy Williamson's paper 'Tennant on Knowable Truth', *Ratio*, 2000; the present author's 'Is every truth knowable? Reply to Williamson', *Ratio*, 2001; and Williamson's response 'Tennant's Troubles', 2007 forthcoming (henceforth: TT).

It would appear that several, if not many, informants had been of the opinion that Tennant (2001b) had effectively disposed of Williamson's original criticisms. For in TT Williamson writes:

> For some time I thought that the problems with [Tennant's] 2001 reply were sufficiently evident to make any further response from me unnecessary. Later experience has taught me otherwise.

In the same anthology as TT is the present author's 'Revamping the Restriction Strategy', written without knowledge of 'Tennant's Troubles' and unrevised in light of it (or them). 'Revamping . . . ' slightly modifies the proposed method of restriction. The modified method circumvents the current disagreement between Williamson and the present author. How it does so will emerge towards the end of this discussion—but only after a rebuttal has been given of Williamson's renewed criticism of the *original* method of restriction proposed in *The Taming of The True*. It is worth seeing how Williamson's criticisms fail even in application to the original method. It is also useful to have this stimulus to re-visit Williamson's alleged Fitchian *reductio* of the restricted Knowability Principle. For the modified restriction thwarts these Fitchian tinkerings, and renders Williamson's quibbles moot.

TT undertakes to clarify the exact construal that Williamson had intended for a certain name 'n' that he had concocted. Tennant (2001b) had engaged Williamson's critique on a different understanding of that name, to be sure. But that was a charitable construal, one based on the assumption that Williamson must have been making a *subtle* error, not a blatant one. Subsequent experience, however, has taught the present author otherwise. Williamson is now insisting on an intended construal of 'n' that makes his error *blatant*. At least, this is how matters appear to stand, to the present author, given Williamson's reply. The blatant error in question will be revealed shortly.

TT does little more than re-state some doctrinal preferences, about knowledge, that one is free to reject.

Finally, TT fails to appreciate the fact that an obvious modification of the definition of Cartesian propositions leaves the restricted Knowability Principle immune to Williamson's line of attack, while still serving its key philosophical purpose for the anti-realist.

Now to details.

3 Williamson's alleged Fitchian reductio of the restricted Knowability Principle

Williamson (2000) furnished an alleged Fitchian reductio of the restricted Knowability Principle. It involves instances of the rule ($\Diamond KC$) for two allegedly Cartesian propositions of the respective forms $\varphi \wedge (K\varphi \rightarrow \theta)$ and $\varphi \wedge (K\varphi \rightarrow \neg\theta)$. In the example that Williamson gives, both φ and θ happen to be decidable propositions—though, as it happens, this feature is not essential to Williamson's argument. The proposition θ, moreover, has the peculiar property that the inferences

$$\frac{\Diamond\theta}{\theta} \quad \text{and} \quad \frac{\Diamond\neg\theta}{\neg\theta}$$

must both be valid. (Such propositions are what the present author called *polar* propositions. Examples are propositions of arithmetic.)

Williamson's choice for φ is 'There is a fragment of Roman pottery at that spot', where a suitable context is assumed so that the referent of 'that spot' is fixed.

Williamson's choice for θ is 'n is even', abbreviated En. Here is how this choice for θ was introduced in Williamson (2000):

> Introduce a name 'n' by the stipulation that it is to designate (rigidly) the number of books actually now on my table. Let E be the predicate 'is even'.

In the rest of his discussion, Williamson used the English rendering 'n is even' for the formal sentence En.[1]

Williamson's two allegedly Cartesian propositions are of the forms

$$\varphi \wedge (K\varphi \rightarrow En) \quad \text{and} \quad \varphi \wedge (K\varphi \rightarrow \neg En).$$

In order to establish the Cartesian status of the first proposition, he offers a story in which an agent knows both that φ and that En. The story is supposed to support the consistency claim

$$K(\varphi \wedge En) \nvdash \bot.$$

[1]Rigidity of names, especially in epistemic contexts, is a contentious issue. But we shall go so far as to grant Williamson his Kripkean presuppositions on this score, as a background to the present dialectic.

The Cartesian status of the second proposition, according to Williamson, would likewise be established by a story in which the agent knows both that φ and that $\neg En$. The second story is supposed to support the consistency claim

$$K(\varphi \wedge \neg En) \nvdash \bot .$$

We shall now reveal the logical structure of Williamson's *reductio* argument, by building it up in stages.

Recall that $K\varphi$ is short for $\exists x \exists t K_t x \varphi$, i.e. some agent at some time knows that φ. Suppose $K(\varphi \wedge \psi)$. Then by the following proof Ω_ξ we may conclude that $K(\varphi \wedge (\xi \rightarrow \psi))$, where ξ is an arbitrary proposition:

$$\Omega^{\xi} \qquad \dfrac{K(\varphi\wedge\psi) \qquad \dfrac{\dfrac{\dfrac{\overline{aK_t(\varphi\wedge\psi)}^{(2)}}{aK_t\varphi} \quad \dfrac{\overline{aK_t(\varphi\wedge\psi)}^{(2)}}{aK_t\psi} \quad \dfrac{\overline{\varphi}^{(1)} \quad \dfrac{\overline{\psi}^{(1)}}{\xi\rightarrow\psi}}{\varphi\wedge(\xi\rightarrow\psi)}}{aK_t(\varphi\wedge(\xi\rightarrow\psi))}{\scriptstyle (1)}}{K(\varphi\wedge(\xi\rightarrow\psi))}}{K(\varphi\wedge(\xi\rightarrow\psi))}{\scriptstyle (2)\ \exists\exists\text{-}E}$$

We are free to substitute any proposition we like for ξ. So let us substitute $K\varphi$ (say) for ξ. We thereby obtain

$$\Omega^{K\varphi} \qquad \dfrac{K(\varphi\wedge\psi) \qquad \dfrac{\dfrac{\dfrac{\overline{aK_t(\varphi\wedge\psi)}^{(2)}}{aK_t\varphi} \quad \dfrac{\overline{aK_t(\varphi\wedge\psi)}^{(2)}}{aK_t\psi} \quad \dfrac{\overline{\varphi}^{(1)} \quad \dfrac{\overline{\psi}^{(1)}}{K\varphi\rightarrow\psi}}{\varphi\wedge(K\varphi\rightarrow\psi)}}{aK_t(\varphi\wedge(K\varphi\rightarrow\psi))}{\scriptstyle (1)}}{K(\varphi\wedge(K\varphi\rightarrow\psi))}}{K(\varphi\wedge(K\varphi\rightarrow\psi))}{\scriptstyle (2)\ \exists\exists\text{-}E}$$

Now let us abbreviate by Π_ψ the resulting argument-for-consistency of

$$K(\varphi\wedge(K\varphi\rightarrow\psi)),$$

i.e. the argument for the Cartesian status of $\varphi\wedge(K\varphi\rightarrow\psi)$. The argument

Π_ψ rests on the claim

$$K(\varphi \wedge \psi) \text{ is consistent (formally: } K(\varphi \wedge \psi) \nvdash \bot)$$

and on the little proof $\Omega^{K\varphi}$ just given, which shows that

$$K(\varphi \wedge \psi) \ \vdash \ K(\varphi \wedge (K\varphi \rightarrow \psi)).$$

As a schematic proof, suitable for embedding within longer proofs, the argument Π_ψ accordingly has the form

$$\Pi_\psi \qquad \dfrac{\dfrac{\begin{array}{c}\Omega^{K\varphi}\\ K(\varphi \wedge \psi) \vdash K(\varphi \wedge (K\varphi \rightarrow \psi))\end{array} \qquad \dfrac{}{K(\varphi \wedge (K\varphi \rightarrow \psi)) \vdash \bot}{\scriptstyle (1)}}{K(\varphi \wedge \psi) \vdash \bot} \qquad K(\varphi \wedge \psi) \nvdash \bot}{\dfrac{\bot}{K(\varphi \wedge (K\varphi \rightarrow \psi)) \nvdash \bot}{\scriptstyle (1)}}$$

The complex knowledge-claim $K(\varphi \wedge (K\varphi \rightarrow \psi))$ implies ψ, as is evident from the following proof Φ_ψ:

$$\Phi_\psi \qquad \dfrac{\dfrac{K(\varphi \wedge (K\varphi \rightarrow \psi))}{K\varphi} \qquad \dfrac{\dfrac{K(\varphi \wedge (K\varphi \rightarrow \psi))}{\varphi \wedge (K\varphi \rightarrow \psi)}}{K\varphi \rightarrow \psi}}{\psi}$$

Note that in this proof the occurrence of $K\varphi$ within $K(\varphi \wedge (K\varphi \rightarrow \psi))$ is crucial. One cannot derive ψ from $K(\varphi \wedge (\xi \rightarrow \psi))$.

We are now ready to construct a 'proof' (call it Σ^ψ) of ψ from the three premises

$$\varphi\ ; \quad \neg K\varphi\ ; \quad K(\varphi \wedge \psi) \nvdash \bot$$

and using both the rule $(\Diamond KC)$ *and* the rule $\dfrac{\Diamond\psi}{\psi}$ (whose use will be justified just in case ψ is polar):

$$
\Sigma^{\psi} \qquad
\dfrac{
  \dfrac{
    \varphi \qquad
    \dfrac{\dfrac{\neg K\varphi \quad \overline{K\varphi}^{\,(1)}}{\bot}}{K\varphi \rightarrow \psi}\,(1)
  }{\varphi \wedge (K\varphi \rightarrow \psi)}
  \qquad
  \begin{array}{c} K(\varphi \wedge \psi) \nvdash \bot \\ \Pi_{\psi} \\ K(\varphi \wedge (K\varphi \rightarrow \psi)) \nvdash \bot \end{array}
}{\Diamond K(\varphi \wedge (K\varphi \rightarrow \psi))}\,(\Diamond KC)
\qquad
\begin{array}{c} \overline{K(\varphi \wedge (K\varphi \rightarrow \psi))}^{\,(2)} \\ \Phi_{\psi} \\ \psi \end{array}
$$

$$
\dfrac{\dfrac{\Diamond K(\varphi \wedge (K\varphi \rightarrow \psi)) \qquad \psi}{\Diamond \psi}\,(2)}{\psi}
$$

Note that it is crucial that one be able to perform the final step of this ‘proof’, from $\Diamond\psi$ to ψ, in order to be able to proceed with the following further considerations.

The formal regimentation given here is faithful to Williamson’s informal argument. We are simply regimenting his informal argument as a natural deduction within epistemic logic.

We are now invited to take the foregoing ‘proof’ Σ^{ψ} twice over, once with Williamson’s polar proposition θ in place of ψ, and once with the polar proposition $\neg\theta$ in place of ψ. The first of these substitutions yields a ‘proof’ Σ^{θ} of θ from the three premises indicated:

$$
\begin{array}{c}
\underbrace{\varphi \;\;,\;\; \neg K\varphi \;\;,\;\; K(\varphi \wedge \theta) \nvdash \bot} \\
\Sigma^{\theta} \\
\theta
\end{array}
$$

and the two rules just mentioned. The second of these substitutions yields a ‘proof’ $\Sigma^{\neg\theta}$ of $\neg\theta$ from the three premises indicated:

$$
\begin{array}{c}
\underbrace{\varphi \;\;,\;\; \neg K\varphi \;\;,\;\; K(\varphi \wedge \neg\theta) \nvdash \bot} \\
\Sigma^{\neg\theta} \\
\neg\theta
\end{array}
$$

and the same two rules. These two ‘proofs’ can then be combined for a final step of $\neg$-Elimination:

$$\dfrac{\dfrac{\underbrace{\varphi\ ,\ \neg K\varphi\ ,\ K(\varphi\wedge\theta)\nvdash\bot}_{\Sigma^{\theta}}}{\theta}\qquad\dfrac{\underbrace{\varphi\ ,\ \neg K\varphi\ ,\ K(\varphi\wedge\neg\theta)\nvdash\bot}_{\Sigma^{\neg\theta}}}{\neg\theta}}{\bot}$$

to give us a 'proof' of $\bot$ from the *four distinct* premises

$$\varphi\ ;\quad \neg K\varphi\ ;\quad K(\varphi\wedge\theta)\nvdash\bot\ ;\quad K(\varphi\wedge\neg\theta)\nvdash\bot.$$

One must not lose sight of the fact that the third and fourth of these are indeed undischarged premises of the argument. In order to regard oneself as hereby possessed of a genuine *reductio ad absurdum* of $\{\varphi, \neg K\varphi\}$, it is essential that one be assured both that

1. $K(\varphi\wedge\theta)\nvdash\bot$ and $K(\varphi\wedge\neg\theta)\nvdash\bot$; and that

2. the two inferences $\dfrac{\Diamond\theta}{\theta}$ and $\dfrac{\Diamond\neg\theta}{\neg\theta}$ are valid.

We shall see, however, that (1) is impossible, in the case where θ is En. This point will stand regardless of Williamson's clarifying insistence in TT that the referent of 'n' is fixed by an *empirical* description. He may fix it how he will, and thereupon use En in place of θ; and still it will not do. The claim (1) is impossible on independent grounds, grounds that Williamson altogether failed to anticipate. Moreover, the notion of deducibility involved in the two consistency claims in (1) can be taken as logico-mathematical. These claims are made good below.

4 Critique of Williamson's alleged Fitchian reductio

4.1 Williamson's name n

The reader of Williamson (2000) could have been forgiven for thinking of 'n' as a placeholder for an unspecified numeral—the numeral that would denote whatever number happened to be picked out by Williamson's description 'the number of books actually now on my table'. This would have been such a natural interpretation for any reader to make that the following measures would have been advisable, especially for the sake of philosophers who are focused on the proper understanding of serious mathematical and scientific investigations.

1. Choose a 'one-off' name free of any of the associations of normal mathematical usage.

2. Stress to the reader that one is departing from the normal conventions in ordinary and scientific discourse, according to which one should be able to say who or what bears a name that one proposes to use.

3. Point out that one is invoking a supposedly successful act of baptism of a number-we-happen-to-know-not—a number to be called by this name '*n*', even if it is a name of a kind that no scientist would ever consider adopting for serious scientific purposes.[2]

4. Point out that the name '*n*', if it really is a rigid designator, would

[2]Matters were very different when Le Verrier introduced the name 'Neptune' to denote the planet that he had conjectured was perturbing the orbit of Uranus. As it happened, Neptune was discovered, and accepted as a new planet, *before* it was named 'Neptune'. Compare Kripke (1980), f. 33 on p. 79: '*If Leverrier* [sic] *indeed gave the name 'Neptune' to the planet before it was ever seen*, then he fixed the reference of 'Neptune' by means of the description ['the planet which caused such and such discrepancies in the orbits of certain other planets'].' [Emphasis added.] History has shown that Kripke's antecedent is false: see Airy (1846), for the Astronomer Royal's meticulous history of published and unpublished sources, revealing how both British and French astronomers and mathematicians hypothesized and finally confirmed the existence of 'the planet exterior to *Uranus*'. In Airy's Royal Astronomical Society Memoir, there is not a single occurrence of the name 'Neptune'. The order of events was: *positing* the existence of 'the' planet exterior to Uranus (one astronomer went so far as to posit the existence of two such planets); *discovery* of such a planet by observation with telescopes; and, only thereafter, *naming* the body whose existence had been confirmed.

Since the case of 'Neptune' is the closest one we can find to that of Williamson's '*n*', it is worth dwelling on some salient differences. First, even under the counterfactual circumstances of Kripke's antecedent, Neptune need not have existed; Le Verrier's conjecture could have been wrong. But the referent of Williamson's '*n*', whatever number it is, is a necessary existent. Secondly, Le Verrier (in the counterfactual circumstances) would have been introducing 'Neptune' as the first name in use for the planet in question. But Williamson's '*n*' could only ever be an *alternative* to some numeral already in use, which denotes the number in question.

appear to be unique among them in not having its referent secured by a baptism at which the witnesses *were able to, and indeed did, identify the bearer of the name*, and were therefore in a position to pass on the name's semantic anchoring, by means of appropriate causal chains connecting them with all other subsequent users of the name.

In Williamson's scenario, the name 'n' is supposed to be secured a referent simply by being associated with a description of the form 'the F'. We have good reason to believe that 'the F' denotes (i.e., we have good reason to believe that there is one and only one F). But no one, it transpires, is ever in a position to say *what particular thing* is the F. Can this really be enough to make 'n' into a rigid designator?

It is a basic constraint on names of natural numbers that their denotations should be effectively determinable, so that one can effectively determine the truth-value of any statement of the form $P(m)$, where m is a numerical name and P is a decidable predicate of numbers. But Williamson's would-be numerical name 'n' is not like that; its adoption would represent a marked departure from established scientific usage.

Contrast how Williamson now introduces his choice of θ in TT. Newly interpolated material is underlined:[3]

> Introduce a <u>proper</u> name 'n' by the stipulation that it is to designate (rigidly) the number of books actually now on my table. <u>Thus 'n' is not a numeral such as '9' but rather a name whose reference is fixed by an empirical description.</u> Let 'E' be the predicate 'is even'.

It is good to have this much-needed clarification of Williamson's intentions. But the result is disastrous. Treating Williamson's 'n' as a proper name for a natural number incurs a direct contradiction with arithmetic.

[3]We ignore the inessential shift from italics in the 2000 paper to boldface in the 2007 paper.

4.2 The direct contradiction with arithmetic

As the foregoing analysis of the logical structure of Williamson's attempted *reductio* has revealed, he needs to claim the following (Williamson's 'n', please note):

1. $K(\varphi \wedge En) \nvdash \perp$; and
2. $K(\varphi \wedge \neg En) \nvdash \perp$.

In order to maintain these two consistency claims, Williamson is relying on his stipulation that *we do not happen to know what the referent of the peculiar name 'n' is.* But since we have $K(\varphi \wedge \psi) \vdash \psi$ in general, it follows by the transitivity of deduction that Williamson needs, consequently, to claim

1′. $En \nvdash \perp$; and

2′. $\neg En \nvdash \perp$.

Now it is a theorem of arithmetic that every natural number is either even or odd. And it is a theorem of arithmetic (upon a suitable coding of syntactic notions)[4] that every natural number is *either provably even or provably odd.* Hence, if Williamson's 'n' really is a name of a natural number, we can conclude that either n is provably even or n is provably odd—*even if we do not know what number the name 'n' denotes.*[5] Suppose, on the one hand,

[4]Note that there is no problem for coding posed by the adoption of Williamson's new name 'n' as a syntactic primitive. One can assign it a code number, and proceed to code complex expressions in the usual way. And one will be able to find a provability predicate for any recursively axiomatized theory in the language of arithmetic extended in this way.

[5]In anticipation of any suspicion that there is a use-mention confusion here, it may be worth explaining how this claim would be regimented. Some writers use Quine's corner-quotes to indicate that a sentence is being mentioned and not used. Other writers systematically suppress corner-quotes, but underline or overline their numerical variables to indicate the presence of the numeral for the number in question. Thus, one might write $\vdash \ulcorner F(ss0) \urcorner$ or write $\vdash F(\underline{2})$, to the same effect. There is also a difference between saying 'it is provable that every number is F' and 'every number is provably F'. The former would be regimented as $\vdash \ulcorner \forall m F(m) \urcorner$, or as $\vdash \forall m F(m)$ (dispensing with the corner-quotes); whereas the latter would be regimented as $\forall m \vdash \ulcorner F(\underline{m}) \urcorner$, or as $\forall m \vdash F(\underline{m})$ (again, dispensing with the corner-quotes).

Let Ex be the formula $\exists k\; k + k = x$. Then we can regiment 'every natural number is provably even or provably odd' as $\forall m(\vdash E\underline{m} \vee \vdash \neg E\underline{m})$. This is a theorem of arithmetic, upon suitable representation of the proof-predicate $\vdash$.

that n is provably even. That contradicts the claim $(2')$ above that it is logico-mathematically consistent to claim that n is not even. Suppose, on the other hand, that n is provably odd. That contradicts the claim $(1')$ above that it is logico-mathematically consistent to claim that n is even. Contradiction.

Williamson, as we have seen, offered a joint *reductio* of the four premises

$$\varphi\ ; \quad \neg K\varphi\ ; \quad K(\varphi \wedge \theta) \nvdash \bot\ ; \quad K(\varphi \wedge \neg\theta) \nvdash \bot.$$

We have just seen that the last pair of these:

$$K(\varphi \wedge \theta) \nvdash \bot\ ; \quad K(\varphi \wedge \neg\theta) \nvdash \bot$$

are jointly (logico-mathematically) inconsistent. Hence there is no reason to take Williamson's *reductio* as establishing the joint inconsistency of the first pair:

$$\varphi\ ; \quad \neg K\varphi\ .$$

The irony should not go unremarked that one of the fruitful applications of the notion of a rigid designator (in modal discourse) is to find a way to avoid opacity problems when instantiating quantifications in modal contexts. Yet Williamson's attempt to cause problems for the anti-realist by means of a supposed rigid designator generates a contradiction with arithmetic right away, upon instantiating an obviously relevant theorem. It would appear that Williamson did not take seriously enough his own claim that 'n' was a *name of a number*.

This point is sufficient to defuse Williamson's renewed critique. But it is worth bringing to light some of the other weaknesses in that critique.

5 The distinction between logical consistency and metaphysical possibility

TT emphasizes the distinction between logical inconsistency and metaphysical impossibility. In his reprise of the semantics of rigid designation, Williamson neglects to draw his reader's attention to the following passage from Tennant (2001b):

> Let us call any proposition θ which, if possibly true, is necessarily true, and, if possibly false, is necessarily false, a polar proposition. ... Examples are 'n is even', or 'n is not even', where 'n' rigidly designates a particular natural number. ... Any provable or refutable mathematical proposition is polar. So too—if we countenance *metaphysical* necessity and impossibility—are propositions such as 'Water is H_2O' and 'Water is XYZ'. Not all polar propositions are effectively decidable. Moreover, if we countenance metaphysical necessity and impossibility, then not all polar propositions' truth-values will be able to be determined *a priori.*
>
> Which propositions are Cartesian in the actual world will depend, in general, on which polar propositions are true. Recall that φ is Cartesian just in case $\neg(K\varphi \vdash \bot)$. To say that absurdity is not derivable from $K\varphi$ is equivalent to saying that absurdity is not derivable from $K\varphi$ in conjunction with any set X of necessarily true propositions. Whether this definition calls for the consideration only of sets X all of whose members are knowable *a priori*, or calls for the consideration also of sets X some of whose members might be knowable only *a posteriori*, is an issue of principle on which we are not at present forced to take a stand. [**fn:** Nowhere in *TToTT* did I claim that the Cartesian character of a proposition would always be an *a priori* matter. But as it happens, Williamson invokes only *a priori* polar propositions, such as 'n is even'. (Bear in mind that this is a *mathematical* proposition, since n is a rigid designator.)][6]

[6]Of course, now that Williamson has clarified the intended construal of 'n', the present author would withdraw the claim that 'n is even' is *a priori.* But that does not help Williamson at all.

It was clear that the present author intended the turnstile in the Cartesian condition $K\varphi \nvdash \perp$ to include use of the axioms and rules of inference of mathematics. How else would one be able to say, for example, that it is impossible simultaneously to know

1. The number of planets is 9; and

2. The number of planets is even ?[7]

From these two propositions one can infer the pure-mathematical proposition that 9 is even—which is *necessarily* false.[8] Hence it is impossible for both (1) and (2) to be true. Hence it is impossible for anyone ever to know their conjunction. Note, however, that both (1) and (2) are *empirical* propositions. But they are inconsistent with one another, in as strong a way (for the purposes of epistemic logic) as that in which the two empirical propositions

1. Some planet in our solar system is more massive than Earth

2. No planet in our solar system is more massive than Earth

are inconsistent with one another.

Williamson accuses the present author of 'startling insensitivity' to the distinction between logical inconsistency and metaphysical impossibility. Yet the true insensitivity is Williamson's, in not detecting the present author's own sensitivity to that very distinction, as displayed in the quote above. Williamson's insensitivity (or lack of charity) is evident again in what he makes of the following passage from Tennant (2001a), at p. 264:

> It should be clear to anyone with a sympathetic understanding of the spirit of the proposed restriction that for a proposition to be Cartesian one ought to be unable to derive absurdity from it

[7] This example was thought up before the International Astronomical Union demoted Pluto on August 24, 2006. But the example still serves its purpose. One does not need to change '9' to '8' and 'even' to 'odd'. For the point concerns impossibility.

[8] By a 'mathematical proposition' here is meant one that is formulated in the language of pure mathematics.

> *modulo* any necessarily true propositions. It is a logical convention of long standing that mention of theorems as premises can be suppressed.

Williamson claims that this passage 'conflates necessary truth and theoremhood'. The present author will now follow Williamson's lead in supplying some extra text to make his intended meaning clear. The final sentence of the passage just quoted should read

> It is a logical convention of long standing that mention of <u>logico-mathematical</u> theorems as premises can be suppressed <u>when exploring the logical relations that hold among contingent statements</u>.

For the anti-realist, there is then no conflation of necessary truth (in mathematics) and theoremhood. This is because for the anti-realist the truth of a mathematical statement consists in its being provable.

It was also clear that the present author took Williamson's example 'n is even' as a mathematical proposition, with 'n' place-holding for whatever numeral denoted the number that happened to be the number of books on Williamson's table at the time he introduced the name in question. Given Williamson's belated clarification of his intentions regarding this name 'n' (which, as we saw earlier, lead immediately to contradiction), it should also be clear, now, that the present author *would* take a stand on the question whether the definition of Cartesian proposition calls for the consideration also of premises that might be knowable only *a posteriori*. The stand is obvious: take them into consideration! Thus, let the turnstile $\vdash$ in the condition $K\varphi \nvdash \perp$ be defined by reference to (suppressed) premises that are *metaphysically* necessary (and also by reference to steps of inference that are *metaphysically*, not just logico-mathematically, guaranteed to preserve truth). Call the resulting turnstile $\Vdash$.[9] So the restricted Knowability Principle, on the more liberal construal of the turnstile, is

[9]This is not to be confused with the notion of forcing in set theory, or with the notion of forcing in Kripke models for intuitionistic logic.

$$(\Diamond KC) \quad \frac{\varphi}{\Diamond K\varphi} \quad \text{where } K\varphi \nVdash \bot$$

Note that the more liberal construal of the turnstile makes it *easier* for a proposition to *fail* to be Cartesian; and therefore makes it *harder* for a proposition to be Cartesian. So the inference in question is safer.

Still, one is at a loss to see why Williamson regards such a re-formulation as 'not distinctively anti-realist', or as 'void[ed] ... of all interest as a formulation of an anti-realist principle of knowability'. Remember that it has all along been conceded, by the present author, that *of course* the Knowability Principle, expressed as a rule of inference, is not a rule for which one has an effective method for checking the correctness of its applications. In some cases, we will be fortunate in being able to say that a particular application is *not* correct—as, for example, with Fitch's original 'proof'. So any ignorance on our part concerning what is metaphysically necessary cannot be held against the formulation of a rule that is intended to serve a certain philosophical, as opposed to scientific, purpose. The purpose in question is to stress that *all truths are knowable* by finite creatures like ourselves. Put another way: no truth could possibly elude detection by finite creatures like ourselves, and do so regardless of the investigations we might undertake and regardless of the evidence we might be able to gather. The safer Principle of Knowability is *still* able to distinguish the anti-realist from the realist on the areas of discourse that are the most important: mathematics and the natural sciences. None of this Fitchian footling is going to blunt the bite of the Knowability Principle *there*.

By analogy, the realist should ask himself whether he would give up the principle of Bivalence for declarative sentences upon discovering the Liar paradox 'This sentence is false'. Realists have *not* given up so easily in the face of the Liar. So why should anti-realists give up easily in the face of the Fitch paradox? With both these paradoxes, something reflexively rum is going on, and one has to cordon off the trouble. The realist does that by (say) stratifying his language, thereby avoiding semantic closure.[10] So henceforth the realist will maintain Bivalence for suitably stratified sentences. And maintaining Bivalence even on this restricted domain of sentences still has enormous philosophical significance. For it might lead (for example) to a justification of the full resources of classical logic in both mathematics and the natural sciences. Likewise, on behalf of the anti-realist, the present author suggested restricting the Knowability Principle to Cartesian propositions. The restricted Knowability Principle still has enormous philosophical significance. For it might lead (for example) to eschewing some of the resources of classical logic in both mathematics and the natural sciences, and to investigating just how much of that logic is actually *needed* for natural science.[11]

As will be shown below, however, the anti-realist will not have to take the stand indicated earlier on $\Vdash$ (in place of $\vdash$). This is because we can re-locate the Cartesian test (to be discussed below) from the immediate premise for the knowability inference, to the set of assumptions on which that immediate premise itself depends. And that will put an end to Williamson's search for a *reductio*, whether he visits it upon the anti-realist by means of his problematic choice of En for θ, or takes for θ instead a metaphysically impossible but logically consistent statement such as the familiar 'Hesperus $\neq$ Phosphorus', or Williamson's own 'George W. Bush = Tony Blair.

Indeed, it is puzzling, in retrospect, why Williamson used En as his choice for θ rather than *any* identity '$a = b$' , where a and b are rigid designators. The latter choice would have forced much earlier the resort to the metaphysical $\Vdash$ in place of the logico-mathematical $\vdash$ in the definition of Cartesian propositions. Williamson could have argued that

1. $K(\varphi \wedge a = b) \nvdash \bot$ (because, among other things, $a = b \nvdash \bot$);
2. $K(\varphi \wedge a \neq b) \nvdash \bot$ (because, among other things, $a \neq b \nvdash \bot$);
3. $\Diamond a = b$ entails $a = b$ (because a and b are rigid designators);

[10] This is not the place to get mired in the many different responses of late to the logical and semantical paradoxes; my choice of response is merely representative, for the purposes of making a convincing analogy.

[11] For further investigation of this question, see Tennant (2006).

4. $\Diamond a \neq b$ entails $a \neq b$ (because a and b are rigid designators).

Thus Williamson's Fitchian *reductio* would have enjoyed the same form as before, only with $a = b$ in place of En.

Consider now how the proponent of the restricted Knowability Principle would react to this *reductio*, given that he is interpreting the turnstile in the definition of Cartesian propositions as $\Vdash$ rather than $\vdash$. The earlier analysis, applied to the new *reductio* involving '$a = b$', would reveal that it depends on four distinct premises:

$$\varphi\ ; \quad \neg K\varphi\ ; \quad K(\varphi \wedge a = b) \nVdash \bot\ ; \quad K(\varphi \wedge \neg a = b) \nVdash \bot.$$

And our critique of that *reductio* would take the same form. Note that we have $K(\varphi \wedge \psi) \vdash \psi$ in general, as well as the principle

$$\frac{\chi \vdash \psi \qquad \psi \Vdash \bot}{\chi \Vdash \bot}\ .$$

Taking $K(\varphi \wedge \psi)$ for χ, and $a = b$ for ψ, it is clear that in order to maintain the truth of the third and fourth premises Williamson needs in this instance to claim

1′. $a = b \nVdash \bot$; and

2′. $\neg a = b \nVdash \bot$.

But it is a theorem of modal logic that $\forall x \forall y(\Box x = y \vee \Box \neg x = y)$. Hence (1′) and (2′) cannot both be true.

So, to summarize the dialectic to this point: Williamson's use of his peculiar name 'n' and the claim En for θ in the schematic argument laid out above poses no counterexample to the original restriction strategy, *even if* the notion of consistency (or possibility) in the definition of Cartesian propositions is logico-mathematical, and not metaphysical. This is because the name 'n' can be used to instantiate a certain universally quantified theorem of arithmetic. *However*: if we employ the schematic argument laid out above with an identity like 'Hesperus = Phosphorus' in place of θ, it may have some purchase, as already conceded. But in that case the obvious thing to do is to accept that the notion of logico-mathematical consistency (or possibility) in the definition of Cartesian propositions has to be replaced by the more demanding notion of metaphysical possibility. Even then, however, there is no damage to the anti-realist's cause. The anti-realist, like any other epistemologist and metaphysician, has to come to terms with Kripke's *a priori* but contingent truths and with his his necessary but *a posteriori* ones.

We turn now to consider how to avoid altogether the kind of problem that Williamson has sought to raise for the restricted knowability principle. Note that the principle ($\Diamond KC$) has imposed the Cartesian restriction 'locally', on the premise φ for the inferential step to the conclusion $\Diamond K\varphi$. It is time now to consider imposing it more 'globally'. It turns out that if we do so, the notion of consistency (or possibility) in the definition of Cartesian propositions can remain logico-mathematical, and does not have to become metaphysical. The next section explains why.[12]

6 The modified restriction strategy circumvents Williamson's alleged Fitchian *reductio*

As mentioned at the outset, the present author's paper 'Revamping the Restriction Strategy', which appears in the same anthology as TT, proposes a (slight) modification of the restriction strategy advocated in *The Taming of The True*. The spirit of the restriction is the same, but the details of its execution are slightly different. The proposal is that the Cartesian restriction on the anti-realist's knowability principle 'φ, therefore $\Diamond K\varphi$' should be formulated as a consistency requirement not on the premise φ of an application of the rule, but rather on the set of assumptions on which the relevant occurrence of φ depends (via whatever proof Π has been constructed). If we limit ourselves to the 'local' restriction, we ignore the contribution of the set Δ of assumptions, focusing instead on the fact that, via Π, we have just reached the conclusion φ, whatever our starting point might have been:

[12] I am grateful to Nicholaos Jones for prompting this clarification of the dialectical position at this juncture.

$$\begin{array}{c}\Delta\\ \Pi\\ \underline{\varphi}\\ \Diamond K\varphi\end{array}\quad \text{where } K\varphi \nvdash \bot$$

But if our grounds for φ are indeed Δ, then the inferred possibility of knowing that φ surely presupposes the possibility of knowing that Δ. Indeed, if it were impossible to know the joint truth of the assumptions in Δ, how could one be confident in inferring from the intermediate conclusion φ to the knowability claim $\Diamond K\varphi$? These considerations lead to the thought that the restriction strategy, instead of looking *down* at φ within Π should rather look *up* at Δ. The proposal, then, is that the restricted Knowability Principle should take the form of the following rule of inference, with a rather more exigent pre-condition for its applicability:

Globally Restricted Knowability Principle

$$\begin{array}{c}\Delta\\ \Pi\\ \underline{\varphi}\\ \Diamond K\varphi\end{array}\quad \text{where } K\Delta \nvdash \bot$$

Here $K\Delta$ is defined in the usual Frobenian way as $\{K\delta \mid \delta \in \Delta\}$. When $K\Delta \nvdash \bot$, we shall say that Δ is Cartesian.

Armed with the Globally Restricted Knowability Principle, the anti-realist can readily dispose of Williamson's alleged Fitchian *reductio*. For now Williamson's attempted applications of ($\Diamond KC$) patently violate the restriction that the set Δ (at that stage) be Cartesian, *even when Cartesian status is construed by reference to logical, not metaphysical, consistency.* For the set in question is precisely $\{\varphi, \neg K\varphi\}$, as the reader can easily verify by inspection. The relevant chunk of Williamson's proof Σ is:

$$(\Diamond KC)\;\dfrac{\dfrac{\varphi \qquad \dfrac{\dfrac{\neg K\varphi \quad \overline{K\varphi}^{\,(1)}}{\bot}}{K\varphi\rightarrow\psi}{\scriptstyle(1)}}{\varphi\wedge(K\varphi\rightarrow\psi)} \qquad \begin{array}{c} K(\varphi\wedge\psi)\nvdash\bot \\ \Pi_{\psi} \\ K(\varphi\wedge(K\varphi\rightarrow\psi))\nvdash\bot \end{array}}{\Diamond K(\varphi\wedge(K\varphi\rightarrow\psi))}\;\text{‘local’}$$

But now the Cartesian condition on the right is supposed to concern not the left premise for the application of $(\Diamond KC)$, but rather the assumptions on which that left premise depends. So this chunk of proof would, *per impossibile*, need to read as follows:

$$(\Diamond KC)\;\dfrac{\dfrac{\varphi \qquad \dfrac{\dfrac{\neg K\varphi \quad \overline{K\varphi}^{\,(1)}}{\bot}}{K\varphi\rightarrow\psi}{\scriptstyle(1)}}{\varphi\wedge(K\varphi\rightarrow\psi)} \qquad \begin{array}{c} \vdots\,? \\ K\varphi, K\neg K\varphi\nvdash\bot \end{array}}{\Diamond K(\varphi\wedge(K\varphi\rightarrow\psi))}\;\text{‘global’}$$

But this will not do. For now the condition $K\varphi, K\neg K\varphi \nvdash \bot$ stated on the right for the would-be application of $(\Diamond KC)$ is patently false, as shown by the two-step proof

$$\dfrac{K\varphi \qquad \dfrac{K\neg K\varphi}{\neg K\varphi}}{\bot}$$

The Globally Restricted Knowability Principle therefore renders Williamson's quibbles moot—whether he raises them with reference to En, or with reference to $a = b$ (where a and b are rigid designators).

References

G. B. Airy. Account of some circumstances historically connected with the discovery of the planet exterior to Uranus. *Monthly Notices of the Royal*

Astronomical Society, VII (9, Nov. 13, 1846):121–144, 1846.

Saul Kripke. *Naming and Necessity (revised and enlarged edition)*. Basil Blackwell Publisher, Oxford, 1980.

Neil Tennant. *The Taming of The True*. Oxford University Press, 1997.

Neil Tennant. Is every truth knowable? Reply to Hand and Kvanvig. *Australasian Journal of Philosophy*, 79(1):107–113, 2001a.

Neil Tennant. Is every truth knowable? Reply to Williamson. *Ratio*, 14(3): 263–280, 2001b.

Neil Tennant. Logic, Mathematics, and the Natural Sciences. In Dale Jacquette, editor, *Philosophy of Logic*, pages 1149–1166. North-Holland, Amsterdam, 2006.

Neil Tennant. Revamping the restriction strategy. In Joseph Salerno, editor, *Every Truth is Known: New Essays on Knowability*, pages x–x. Oxford University Press, Oxford, 2007.

Timothy Williamson. Tennant on knowable truth. *Ratio*, 13(2):99–114, 2000.

Timothy Williamson. Tennant's Troubles. In Joseph Salerno, editor, *Every Truth is Known: New Essays on Knowability*, pages x–x. Oxford University Press, Oxford, 2007.