

The Philosophical Quarterly

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BLACKWELL PUBLISHING
FOR
THE SCOTS PHILOSOPHICAL CLUB
AND THE
UNIVERSITY OF ST ANDREWS

RULE-CIRCULARITY AND THE JUSTIFICATION OF DEDUCTION

BY NEIL TENNANT

I examine Paul Boghossian's recent attempt to argue for scepticism about logical rules. I argue that certain rule- and proof-theoretic considerations can avert such scepticism. Boghossian's 'Tonk Argument' seeks to justify the rule of tonk-introduction by using the rule itself. The argument is subjected here to more detailed proof-theoretic scrutiny than Boghossian undertook. Its sole axiom, the so-called Meaning Postulate for tonk, is shown to be false or devoid of content. It is also shown that the rules of Disquotatation and of Semantic Ascent cannot be derived for sentences with tonk dominant. These considerations deprive Boghossian's scepticism of its support.

I. INTRODUCTION

My point of departure is a recent paper by Paul Boghossian.¹ Boghossian tries to get his readers to entertain sceptical doubt as to whether certain introduction and elimination rules of inference in propositional logic are justifiable. Is his scepticism worth taking seriously (for as long as he himself took it, before disposing of it)? I shall argue that it is not.

It is well known that the usual logical rules can be shown to be truth-preserving (only) by means of arguments which use those very rules. The quietist or objectivist regards this fact as the inevitable consequence of those rules' being, epistemologically speaking, absolutely fundamental, or primitive. Whatever it is about logic that is to be targeted by sceptical doubts, one must be permitted and able to use at least some of that logic in order to deal with those doubts.²

¹ P. Boghossian, 'How are Objective Epistemic Reasons Possible?', *Philosophical Studies*, 106 (2001), pp. 1–40; see also Crispin Wright's commentary, 'On Basic Logical Knowledge: Reflections on Boghossian's "How are Objective Epistemic Reasons Possible?"', *Philosophical Studies*, 106 (2001), pp. 41–85.

² As Thomas Nagel trenchantly puts it, 'Certain forms of thought can't be intelligibly doubted because they force themselves into every attempt to think about anything': T. Nagel, *The Last Word* (Oxford UP, 1997), p. 61.

There has been an unspoken assumption, however, to the effect that the rule-circularity of justifications of logical rules is not vicious, on the ground that *only* the *usual* logical rules can be justified in this way. Boghossian's strategy, in justifying scepticism about the usual rules of logic, is to establish what he regards as a perfect analogy: a certain *deviant* rule can be shown to be truth-preserving, by means of an argument that uses that very rule. This supposed analogy between the standard case and the deviant case challenges the unspoken assumption just mentioned, and raises sceptical doubts about the usual rules – so Boghossian contends.

I shall argue that Boghossian's analogy is imperfect. His arguments are open to decisive objections. In the light of these objections, the arguments in question can provide no support at all for scepticism about the usual rules of logic.

There are two kinds of deviant rule to be considered. First, there are rules for deviant connectives, such as Arthur Prior's 'tonk'.³ Secondly, there are deviant rules for standard connectives (such as Denying the Antecedent, for the conditional). Boghossian focuses on the former, Wright on the latter. I have space enough here only to deal with the former; but I can assure the reader that the principles that I shall adduce will dispose also of the latter.

II. THE TONK ARGUMENT

Playing devil's advocate for the tonk-user, Boghossian presents the following argument, which he attributes to Wright. I shall call it 'the Tonk Argument' in what follows.⁴

The Tonk Argument

- | | | |
|----|--|---------------------------------|
| 1. | ' P tonk Q ' is true iff
' P is true tonk ' Q ' is true | Meaning Postulate |
| 2. | P | Assumption |
| 3. | ' P is true | 2, T -Scheme |
| 4. | ' P is true tonk ' Q ' is true | 3, tonk introduction |
| 5. | ' P tonk Q ' is true | 4, 1, biconditional elimination |
| 6. | P tonk Q | 5, T -Scheme |
| 7. | If P , then P tonk Q | 2–6, conditional introduction |

Some abbreviations will make matters clearer. I shall use $T()$ for the truth predicate, and \rightarrow and \leftrightarrow for the conditional and biconditional, respectively.

³ See A.N. Prior, 'The Runabout Inference Ticket', *Analysis*, 21 (1960), pp. 38–9.

⁴ I have supplied the line numbers, which were clearly intended, but happened to be omitted in Boghossian's text.

I shall suppress the single quotation marks around P , Q , etc. I shall abbreviate *tonk* to t . The Tonk Argument above can then be rendered as the following Gentzen–Prawitz-style natural deduction. I choose this form of regimentation because it makes logical dependencies very clear to the eye.

The Tonk Argument as a Gentzen–Prawitz-style Natural Deduction

$$\frac{\frac{\frac{\overline{P}^{(1)}}{T(P)^{(TI)}}}{T(P) \ t \ T(Q)^{(tI)}} \quad \text{‘Meaning Postulate’: } T(P \ t \ Q) \leftrightarrow [T(P) \ t \ T(Q)]}{\frac{T(P \ t \ Q)}{P \ t \ Q}^{(TE)}}^{(\leftrightarrow E)} \quad \frac{P \ t \ Q}{P \rightarrow (P \ t \ Q)}^{(1)(\rightarrow I)}$$

The following observations about this argument should be uncontentious.

1. The Tonk Argument seeks to justify the rule of tonk-Introduction by using the same rule.
2. The conclusion of the Tonk Argument is the statement $P \rightarrow (P \ t \ Q)$. This involves no occurrences of the truth predicate. (Presumably this is the sought statement to the effect that the rule of tonk-Introduction is justified.)
3. The arguer uses the rule of tonk-Introduction. But the arguer does not use the rule of tonk-Elimination.
4. The Tonk Argument is intended to be *a priori*. In particular, the ‘Meaning Postulate’ $T(P \ t \ Q) \leftrightarrow [T(P) \ t \ T(Q)]$ (at least in the right–left direction) is supposed to be *a priori*. The Tonk Argument treats it as an axiom on which the conclusion $P \rightarrow (P \ t \ Q)$ can be made to rest. The two halves of the T -scheme, namely, the inference rules

$$\frac{\phi}{T(\phi)} \quad (T\text{-Introduction, or Semantic Ascent})$$

$$\frac{T(\phi)}{\phi} \quad (T\text{-Elimination, or Disquotation})$$

are likewise *a priori*, as are the rules of \rightarrow -Introduction and \leftrightarrow -Elimination.

III. HARMONY

Any introduction rule, taken on its own, succeeds in conferring on its featured connective a precise logical sense. That sense in turn dictates what

the corresponding elimination rule must be. *Mutatis mutandis*, any elimination rule, taken on its own, succeeds in conferring on its featured connective a precise logical sense. That sense in turn dictates what the corresponding introduction rule must be.

The notion of harmony for logical operators goes back to the seminal writings of Gerhard Gentzen, who laid the foundations for modern proof theory.⁵ Gentzen's great insight was that

every purely logical proof can be reduced to a definite, though not unique, normal form. Perhaps we may express the essential properties of such a normal proof by saying: it is not roundabout. No concepts enter into the proof other than those contained in its final result, and their use was therefore essential to the achievement of that result.

In order to prove this normalization result, the so-called *Hauptsatz*, for any logical system,⁶ the proof-theorist makes essential use of the fact that there is a certain constitutive balance between the introduction rule for a logical operator and its associated elimination rule.⁷ This is the upshot of Nuel Belnap's famous reply to Prior's original paper on tonk, in reference to 'right' and 'left' sequent rules;⁸ and it was captured again three years later by Prawitz's well known 'Inversion Principle' for the introduction and elimination rules of natural deduction.

In the natural-deduction setting the constitutive balance can be described as follows. The introduction rule states the conditions under which a conclusion with that operator dominant can be inferred. It describes, if you like, the obligations that have to be met by the speaker in order to be justified in asserting the conclusion in question.⁹ The corresponding elimination rule states what the listener is entitled to infer from the speaker's assertion.

⁵ See G. Gentzen, 'Untersuchungen über das logische Schliessen', *Mathematische Zeitschrift*, 1 (1935), pp. 175–210, tr. as 'Investigations into Logical Deduction', in *The Collected Papers of Gerhard Gentzen*, ed. M.E. Szabo (Amsterdam: North-Holland, 1969), pp. 68–131.

⁶ In the sense in which Gentzen intended it, a '*Hauptsatz*' would be a result that explicated this informal claim just quoted. It is mere historical accident that the term has come to be associated with the Cut-Elimination Theorem for a *sequent* system. For reasons which I do not have space to explain or examine here, Gentzen did not achieve a *Hauptsatz* for his systems of natural deduction. That breakthrough was made by Prawitz, some three decades later. See my 'Ultimate Normal Forms for Parallelized Natural Deductions', *Logic Journal of the IGPL*, 10(3) (2002), pp. 1–39, for a much fuller discussion of the historical and proof-theoretic issues involved in perfecting a *Hauptsatz* for systems of natural deduction – systems that are appropriately 'parallelized'.

⁷ Or, in the terminology of sequent systems: between the rule for introducing a logical operator into the *succedent* of a sequent ('on the right'), and the rule for introducing it into the *antecedent* of a sequent ('on the left').

⁸ N. Belnap, 'Tonk, Plonk and Plink', *Analysis*, 22 (1962), pp. 130–4.

⁹ Strictly speaking, this remark applies to *canonical* arguments whose last steps are introductions. But there is no loss of generality; on the contrary, there is a well worked out theory of how the validity of arguments in general reduces to the validity of canonical

Clearly, these mutual obligations and entitlements have to be in balance. The listener must not be allowed to infer more than is required to have been 'put in' to the justification of the assertion. Likewise, the speaker must not be able to get away with less than is required when one is giving the listener entitlement to make certain inferences from what has been asserted. This way of putting matters, stressed with far-reaching consequences by Michael Dummett,¹⁰ reckons the meanings of logical operators to be importantly tied to the norms inherent in the 'social contract' of language-use. It prompts the not unreasonable conjecture that no language could have evolved unless its logical operators were subject to harmonious rules for their introduction into conclusions, and subsequent elimination from premises. On this view, harmony for logical operators would be a transcendental constraint on the very possibility of logically structured communication.¹¹

Indeed, a logical word's being governed by harmonious rules is what confers upon it its precise logical sense. The rules have to be in place for the word to mean what it does. There is no independent access to the meaning of the word, with respect to which one could then raise the question whether the rules governing it 'respect' that sense, or properly transmit truth to and from sentences involving it.

These considerations about the matching of speaker obligations to listener entitlements prompt one (rather more holistic) way of explicating

arguments. See D. Prawitz, 'On the Idea of a General Proof Theory', *Synthese*, 27 (1974), pp. 63–77. See also my *Anti-Realism and Logic: Truth as Eternal* (Oxford: Clarendon Press, 1987), ch. 13, 'The Recursive Definition of Valid Argument', pp. 134–43. These treatments stress the introduction rules as sense-constituting, and the elimination rules as sense-explicating. But one can also reverse that order, and take the elimination rules as sense-constituting, with the introduction rules as sense-explicating. This shifts our attention from the role of a logical operator as dominant in a sentence to be proved to its role as dominant in a sentence to be refuted (*modulo* others, in general). For an appropriate recursive definition of the validity of *reductio* arguments, see my *The Taming of the True* (Oxford UP, 1997), ch. 12, 'Defeasibility and Constructive Falsifiability'.

¹⁰ See Dummett's classic articles 'The Justification of Deduction' (1973) and 'The Philosophical Basis of Intuitionistic Logic' (1975), repr. in his *Truth and Other Enigmas* (London: Duckworth, 1978). See also Prawitz's important paper 'Meaning and Proofs: on the Conflict between Classical and Intuitionistic Logic', *Theoria*, 43 (1977), pp. 2–40. Dummett discusses the philosophical implications of Prawitz's ideas further in his *The Logical Basis of Metaphysics* (Harvard UP, 1991).

¹¹ For more detailed discussion of the social contract of language-use, and how harmony of introduction and elimination rules explicates the idea that speakers' inferential obligations should and can be balanced by listeners' inferential entitlements, see my *Anti-Realism and Logic*, ch. 9, 'Harmony and the Aetiology of Entrenchment', pp. 76–98. See also my 'Intentionality, Syntactic Structure and the Evolution of Language', in C. Hookway (ed.), *Minds, Machines and Evolution* (Cambridge UP, 1984), pp. 73–103; 'In Defence of Evolutionary Epistemology', *Theoria*, 49 (1983), pp. 32–48; and 'Logic and its Place in Nature', in P. Parrini (ed.), *Kant and Contemporary Epistemology* (Amsterdam: Kluwer, 1994), pp. 101–13.

harmony, which does not mention deontic matters, but focuses only on issues of logical strength.¹² Call ϕ the *strongest* proposition with property F if and only if ϕ has F and logically implies any proposition that has F . And call ϕ the *weakest* proposition with property F if and only if ϕ has F and is logically implied by any proposition with property F . Then one can say, of any compound $A @ B$ with connective $@$, that

- $A @ B$ is the strongest proposition that can feature as the conclusion of the introduction rule for $@$, given the elimination rule
- $A @ B$ is the weakest proposition that can feature as the major premise of the elimination rule for $@$, given the introduction rule.

Another, rather more local, way of explicating the harmony between the introductory and the eliminative aspects of a logical operator is to be found in the *reduction procedures* that are designed to eliminate from proofs occurrences of sentences standing simultaneously as conclusions of introductions and as major premises of eliminations. The normalization theorem (or *Hauptsatz*) for a system of natural deduction says that such *maximal* sentence occurrences (as they are called) can be eliminated from proofs by finitely many applications of these reduction procedures. The resulting proof is said to be in *normal form*. The striking thing about a proof in normal form is that the reasoning it represents ‘dismantles’ the premises into their constituents and ‘reassembles’ the information therein so as to form the conclusion. The logical passage from premises to conclusion is thereby shown to arise from the *internal constituent structure* of the premises and the conclusion.¹³

IV. INFERENCE TRUTH THEORY

Let us take a closer look at the behaviour of logical connectives, and try to find out what lies behind or beneath their being meaningful. We shall find

¹² See my *Natural Logic* (Edinburgh UP, 1978).

¹³ The *locus classicus* for such normalization theorems is D. Prawitz, *Natural Deduction: a Proof-Theoretical Study* (Stockholm: Almqvist & Wiksell, 1965). Prawitz furnished the missing *Hauptsätze* for various systems of natural deduction (minimal, intuitionistic and classical) albeit for a reduced logical vocabulary in the classical case. The normalization theorem for classical logic with all the usual operators primitive was obtained only much later – see G. Stålmarmark, ‘Normalization Theorems for Full First Order Classical Natural Deduction’, *Journal of Symbolic Logic*, 56(1) (1991), pp. 129–49. Another important figure in philosophically motivated proof theory is Per Martin-Löf, who has concentrated on higher-order and typed systems. See his paper ‘Constructive Mathematics and Computer Programming’, in L.J. Cohen *et al.* (eds), *Logic, Methodology and Philosophy of Science* (Amsterdam: North-Holland, 1982), pp. 153–75, and his book *Intuitionistic Type Theory* (Napoli: Bibliopolis, 1984). As noted earlier, Gentzen had been able to obtain *Hauptsätze* only for sequent systems.

that harmony between their introduction and elimination rules is a key prerequisite for their meaningfulness. I shall begin with the theory of truth.

IV.1. *The standard Tarskian approach*

As noted above, Tarski's biconditional T -Scheme $\phi \leftrightarrow T(\phi)$ can be divided into two halves, each one dealing with one of the directions of the biconditional. Expressed as rules of inference, these are, respectively, as stated above:

$$\frac{\phi}{T(\phi)} \quad (T\text{-Introduction, or Semantic Ascent})$$

$$\frac{T(\phi)}{\phi} \quad (T\text{-Elimination, or Disquotation})$$

Tarski also gave inductive clauses in his definition of truth, one for each logical operator. These too were of biconditional form. In the case of conjunction, the clause is

$$T(A \wedge B) \leftrightarrow [T(A) \wedge T(B)].$$

(We deal here just with truth, not with satisfaction. The case of a propositional language provides sufficient complexity for the purposes of our philosophical discussion.) Staying with conjunction for the purposes of illustration, this definitional clause can likewise be divided into two halves, each half dealing with one of the directions of the biconditional. Expressed as rules of inference, these are, respectively:

Distribution of Truth over Conjunction:

$$\frac{T(A \wedge B)}{T(A) \wedge T(B)}$$

Compounding of Truth over Conjunction:

$$\frac{T(A) \wedge T(B)}{T(A \wedge B)}$$

One way to set out Tarski's theory of truth for a language with conjunction primitive would be to lay down these two rules as primitive, within an inferentially formulated theory of truth. The collection of all such Distribution and Compounding rules for primitive connectives, along with 'basis' rules for atomic sentences

$$\frac{A}{T(A)} \quad \frac{T(A)}{A}$$

will allow one to derive all instances of Disquotation and of Semantic Ascent – that is, to derive all instances of the *T*-Scheme. This indeed is the standard way of formulating truth-theory, and of showing that it is ‘materially adequate’. On this approach, one takes as basic the truth-functional behaviour of the connectives. That behaviour is expressed by the various pairs of rules of Compounding and Distribution of Truth, one pair for each connective. Then one shows that for all sentences ϕ , ϕ is true if and only if ϕ .

IV.2. *An alternative, inferential form of truth-theory*

There is a simple refinement of truth-theory, however, which merits closer consideration here. On this approach, one uses *rules of inference* governing the truth predicate, modelled directly on the rules of natural deduction for the connectives. So, rather than adopting the rules of Distribution and Compounding of Truth as primitive inferences for each connective, one adopts instead what could be called the *truth-predicational variants* of the primitive introduction and elimination rules for those connectives.¹⁴ I shall illustrate the idea with conjunction. Take its usual introduction and elimination rules:

$$\begin{array}{ccc}
 (\wedge\text{-Intro}) & & (\wedge\text{-Elim}) \\
 \frac{A \quad B}{A \wedge B} & & \frac{A \wedge B}{A} \quad \frac{A \wedge B}{B}
 \end{array}$$

The way to obtain the truth-predicational variants of these rules is simply to prefix the compound sentence and its components with the truth predicate. In this fashion we obtain the following rules:

Truth-predicational rules for conjunction:

$$\begin{array}{ccc}
 (T\wedge\text{-Intro}) & & (T\wedge\text{-Elim}) \\
 \frac{T(A) \quad T(B)}{T(A \wedge B)} & & \frac{T(A \wedge B)}{T(A)} \quad \frac{T(A \wedge B)}{T(B)}
 \end{array}$$

The material adequacy of the inferential truth-theory can be established in the usual way, by deriving all instances of Disquotation and of Semantic Ascent. Here is the proof of Disquotation for conjunctions (assuming, by way of inductive hypothesis, that we have Disquotation for the conjuncts). It uses $(\wedge\text{-Intro})$ and $(T\wedge\text{-Elim})$.

¹⁴ Inferential truth-theory is set out in my *Anti-Realism and Logic*, pp. 71–4.

$$\frac{\frac{T(A \wedge B)}{T(A)} \quad \frac{T(A \wedge B)}{T(B)}}{A \quad B} \quad A \wedge B$$

And here is the proof of Semantic Ascent for conjunctions (assuming, by way of inductive hypothesis, that we have Semantic Ascent for the conjuncts). It uses (\wedge -Elim) and ($T\wedge$ -Intro).

$$\frac{\frac{A \wedge B}{A} \quad \frac{A \wedge B}{B}}{\frac{T(A)}{T(B)}} \quad T(A \wedge B)$$

In the standard approach discussed in §IV.1, the Tarskian clause for conjunction was taken as primitive within the inductive definition of truth. But on the approach we are considering here, that clause is now (non-trivially) derived. For it corresponds, in effect, to the two rules of Distribution and Compounding of Truth over conjunction. And those two rules are now (non-trivially) derived. Here is the derivation of Distribution of Truth over conjunction. It uses (\wedge -Intro) and ($T\wedge$ -Elim).

$$\frac{\frac{T(A \wedge B)}{T(A)} \quad \frac{T(A \wedge B)}{T(B)}}{T(A) \wedge T(B)}$$

And here is the derivation of Compounding of Truth over conjunction. It uses (\wedge -Elim) and ($T\wedge$ -Intro).

$$\frac{\frac{T(A) \wedge T(B)}{T(A)} \quad \frac{T(A) \wedge T(B)}{T(B)}}{T(A \wedge B)}$$

Now in a conjunction, both conjuncts count as *positive* immediate subformulae.¹⁵ That is why, in the proof of Semantic Ascent for conjunctions, we appealed, by way of inductive hypothesis, only to Semantic Ascent for the conjuncts. (A similar remark holds with ‘Semantic Ascent’ uniformly replaced by ‘Disquotation’.) By way of illustrative contrast, here is what

¹⁵ Positive and negative subformulae are defined inductively as follows – see my *Autologic* (Edinburgh UP, 1992), p. 68: every formula is a positive subformula of itself. If $\neg A$ is a positive [negative] subformula of B , then \perp is a positive [negative] subformula of B , but A is a negative [positive] subformula of B . If $A \wedge B$ is a positive [negative] subformula of C , then so are A and B ; and similarly for disjunction. If $A \rightarrow B$ is a positive [negative] subformula of C , then B is a positive [negative] subformula of C , but A is a negative [positive] subformula of C .

happens with a compound containing a *negative* immediate subformula. Consider the conditional, with its introduction and elimination rules:

$$\begin{array}{c} (\rightarrow\text{-Intro}) \\ \overline{A}^{(i)} \\ \vdots \\ B \\ \hline A \rightarrow B^{(i)} \end{array} \qquad \begin{array}{c} (\rightarrow\text{-Elim}) \\ A \rightarrow B \quad A \\ \hline B \end{array}$$

The truth-predicational variants of these rules are as follows.

Truth-predicational rules for the conditional:

$$\begin{array}{c} (T\rightarrow\text{-Intro}) \\ \overline{T(A)}^{(i)} \\ \vdots \\ T(B) \\ \hline T(A \rightarrow B)^{(i)} \end{array} \qquad \begin{array}{c} (T\rightarrow\text{-Elim}) \\ T(A \rightarrow B) \quad T(A) \\ \hline T(B) \end{array}$$

These truth-rules for the conditional were obtained in the same simple way as those for conjunction. We prefixed the compound and its components with the truth predicate at every one of their occurrences within the usual introduction- and elimination-rules (here, the rules of Conditional Proof and the rule of *modus ponens*). In the proof of Disquotation for conditionals, we need by way of inductive hypothesis to assume Semantic Ascent for the *negative* subformula A (the antecedent) and Disquotation for the *positive* subformula B (the consequent). This proof also uses ($\rightarrow\text{-Intro}$) and ($T\rightarrow\text{-Elim}$).

$$\frac{\frac{\frac{\overline{A}^{(1)}}{T(A)}}{T(A \rightarrow B)}}{\frac{T(B)}{B}}}{A \rightarrow B^{(1)}}$$

Likewise, in the proof of Semantic Ascent for conditionals, we need, by way of inductive hypothesis, to assume Disquotation for A and Semantic Ascent for B . This proof also uses ($\rightarrow\text{-Elim}$) and ($T\rightarrow\text{-Intro}$).

$$\frac{\frac{A \rightarrow B}{B}}{\frac{\overline{T(A)}^{(1)}}{A}}}{T(A \rightarrow B)^{(1)}}$$

As with conjunction, however, no such inductive hypotheses are needed for the proofs of Distribution and of Compounding. Here is the proof of Distribution of Truth over the conditional. This proof uses (\rightarrow -Intro) and ($T\rightarrow$ -Elim).

$$\frac{\frac{T(A \rightarrow B) \quad \overline{T(A)}^{(1)}}{T(B)}}{T(A) \rightarrow T(B)}^{(1)}$$

And here is the proof of Compounding of Truth over the conditional. This proof uses (\rightarrow -Elim) and ($T\rightarrow$ -Intro).

$$\frac{\frac{T(A) \rightarrow T(B) \quad \overline{T(A)}^{(1)}}{T(B)}}{T(A \rightarrow B)}^{(1)}$$

We see more clearly now how the introductory and eliminative aspects of a meaningful connective like conjunction or the conditional are intertwined. Those aspects are directly registered in the inferential truth-theory whose basic inferences, apart from the introduction and elimination rules for the connectives, are obtained by truth-predicational variation on those introduction and elimination rules themselves.

It is only because the usual connectives have introduction rules and elimination rules which are in harmony with one another, this harmony being respected in their truth-predicational variants, that those connectives turn out to be meaningful. That is, they *provably* obey Compounding and Distribution of Truth. The result is *a priori*, to be sure; but it need not be postulated as a meaning-theoretic axiom. Instead, on an appropriately careful analysis, it can be derived.

(Provably) obeying Compounding and Distribution of Truth is necessary, but not sufficient, for truth-functionality. A connective is truth-functional if, but only if, any assignment of truth-values to the sentences it connects determines a unique truth-value for the compound thereby formed. The intuitionistic connectives are not truth-functional, because no finite matrices for the connectives are characteristic for intuitionistic propositional logic.¹⁶ Nevertheless, the intuitionistic connectives obey Compounding and Distribution of Truth. This is a necessary and sufficient condition to be satisfied before one can set forth a ‘Meaning Postulate’ in the usual form for them. By a well known result of Dugundji, the modal operators likewise fail to

¹⁶ See K. Gödel, *Zum intuitionistischen Aussagenkalkül* (Wien: Anzeiger der Akademie der Wissenschaften in Wien, 1932); English translation in Vol. 1 of *Collected Works*, ed. S. Feferman *et al.* (Oxford UP, 1986), pp. 224–5.

be truth-functional, for any *finite* number of truth-values that might be involved.¹⁷ But the modal operators too obey Compounding and Distribution of Truth.

V. SUMMARY OF PROOF-THEORETIC POINTS

Here now is a summary of the proof-theoretic points made thus far:

1. An introduction rule alone will fix a sense for the dominant logical operator in its conclusion
2. An elimination rule alone will fix a sense for the dominant logical operator in its major premise
3. These senses will be identical if, but only if, those two rules are in harmony
4. Harmonious rules have reduction procedures that enable one to eliminate from a proof any sentence-occurrence standing as the conclusion of an introduction and as the major premise of an elimination
5. The reduction procedures ensure that every proof can be brought into normal form
6. A proof in normal form dismantles its premises and reconstitutes (some of) their parts so as to yield the conclusion
7. Harmony is required for meaningfulness
8. Harmonious rules have truth-predicational variants that enable one to prove that the rules are valid
9. In such a soundness proof, one might need to employ an elimination rule for an operator in order to show that its introduction rule is valid, and *vice versa*.

VI. HOW TONK FARES IN THE LIGHT OF THESE POINTS

We usually deal only with the familiar connectives, equipped with their well known truth-tables. When we do happen to venture beyond the usual ones, and work with, say, the Sheffer stroke, we are still dealing with a connective whose introduction and elimination rules are in harmony. I have given elsewhere introduction and elimination rules for Sheffer's stroke.¹⁸ To recapitulate, they are as follows:

¹⁷ See J. Dugundji, 'Note on a Property of Matrices for Lewis and Langford's Calculi of Propositions', *Journal of Symbolic Logic*, 5 (1940), pp. 150–1.

¹⁸ Tennant, 'La barre de Sheffer dans la logique des sequents et des syllogismes', *Logique et Analyse*, 88 (1979), pp. 503–14.

$$\begin{array}{c}
 \text{\scriptsize (i)} \overline{\quad} \quad \overline{\quad} \text{\scriptsize (i)} \\
 \underbrace{A \quad B} \\
 \vdots \\
 \perp \\
 \hline
 A|B \text{\scriptsize (i)}
 \end{array}
 \qquad
 \text{\scriptsize |-Elim} \quad \frac{A \quad A|B \quad B}{\perp}$$

These rules suffice for the ‘left–right’ reading of the truth-table for Sheffer’s stroke, in the sense explained in my article ‘Truth Table Logic’.¹⁹ In order to obtain classical logic based on Sheffer’s stroke, one needs also a classical ‘rule of negation’, such as classical *reductio ad absurdum*:

$$\frac{\overline{A|A} \text{\scriptsize (i)}}{\vdots}
 \frac{\perp}{A} \text{\scriptsize (i)}$$

Because we tend to work only with connectives that have harmoniously balanced introduction and elimination rules, it is easy to lose sight of the point that harmony of rules is required for meaningfulness. Also, the very existence of a truth-table for a connective γ means that γ will obey both Compounding and Distribution.

In the case of tonk, however, we are dealing with a connective that is supposed to be characterized solely by its inferential rules (for introduction in a conclusion, and for elimination from a major premise). This means that we cannot simply assume as unproblematic the existence of a truth-table for tonk. Indeed, the peculiarity of tonk is that it has no sense. This is because when we look at how tonk has been given to us, we find two would-be rules of inference, an apparent introduction rule and an apparent elimination rule, which are not in harmony. Hence tonk cannot be meaningful. Tonk will fail to obey either Compounding or Distribution (of Truth).

Prior’s inventive mischief was precisely to yoke together an introduction rule and an elimination rule which respectively characterize *different* logical operators. Prior’s introduction rule

$$\frac{A}{A @ B}$$

as observed above, confers on $A @ B$ the meaning ‘ A , regardless whether B ’. By contrast, Prior’s elimination rule

$$\frac{A * B}{B}$$

¹⁹ ‘Truth Table Logic, with a Survey of Embeddability Results’, *Notre Dame Journal of Formal Logic*, 30 (1989), pp. 459–84.

would confer on $A * B$ the meaning ‘Regardless whether A, B ’ (or ‘ B , even if A ’). There is no logical operator that these two rules, taken together, succeed in characterizing, when we have the same dominant operator in the conclusion of the introduction rule as we have in the major premise of the elimination rule. The connective symbol t simply fails to have a sense when forced to obey the pair of rules

$$\frac{A}{A t B} \quad \frac{A t B}{B}$$

A fortiori, t fails to have a logical operator for its sense. This point can escape the unwary who unquestioningly call t a logical connective. Tonk has the grammatical features of a binary connective, to be sure; but it has no sense, no semantic value at all. It is not a genuine logical connective.

This diagnosis parallels the discovery that some non-logical expression is ambiguous between two precise senses which it would be dangerous to conflate. The fallacy of equivocation is the fallacy of taking at least one of the premises involving an ambiguous term on one of its interpretations, and passing, in the course of one’s reasoning, to the other interpretation of that same term when it occurs in the conclusion. The equivocating shift, in the case of tonk, is disastrous in its consequences. For Prior’s introduction and elimination rules respectively confer on the *term* ‘tonk’ two quite irreconcilable interpretations. Tonk fails the proof-theorist’s test for univocity, that its elimination rule should be *in harmony with* the introduction rule.

Hence, anticipating a criticism to be pressed below, because tonk is not meaningful, the ‘Meaning Postulate’ for tonk is no such thing at all. Indeed, Wright himself writes (p. 49) of its being only a ‘(purported)’ meaning postulate. The ‘Meaning Postulate’ for tonk is, at best, a false claim about the alleged meaningfulness of something that looks, grammatically, as though it might be a logical connective. At worst, as Wright suggests (pp. 82–3, fn. 10), it is a claim without content, in so far as it is not just *about* the problematic ‘connective’ in question, but is expressed *by means of it*. Tonk turns out to be no more than a new-fangled ‘grammatical’ device which forms a sentence from two sentences. Its ‘logicality’, despite these appearances, is an illusion.

VII. THE TONK ARGUMENT REVISITED

We are now in a position to remark on certain curious aspects of the Tonk Argument, in the light of the regimentation I have given, the general proof-theoretical points I have made and the philosophical purposes Boghossian intends it to serve.

Boghossian’s imagined ‘practitioners’ of the tonk-rules would not be inferring. This is because inferring, as a rational activity, aims at the transmission of truth. The tonk rules could have no role in this activity, because, notoriously, they fail to transmit truth. Instead, they allow one to ‘infer’ any proposition *B* from any other proposition *A*:

$$\frac{\frac{A}{A \text{ tonk } B} \text{ (tonk-Intro)}}{B} \text{ (tonk-Elim)}$$

Hence they also allow one to prove any proposition *B* outright, from no assumptions whatever. All one needs to be able to do is establish one logical truth, such as $A \rightarrow A$, and the rest is easy:

$$\frac{\frac{\frac{\overline{A}^{(1)}}{A \rightarrow A}^{(1)}}{(A \rightarrow A) \text{ tonk } B} \text{ (tonk-Intro)}}{B} \text{ (tonk-Elim)}$$

So the addition of tonk to a language lacking it results in the trivialization of the deducibility relation on the original language. This shows that tonk cannot be a logical operator.

No one who infers arbitrary propositions from any given premise is rational. Nor is anyone who regards every statement as *a priori* provable. Thus no one is rational who endorses ‘rules’ that can be seen in a flash to enable one so to infer and prove. Indeed, not even a dialetheist, one who believes that there are true contradictions, will go so far as to claim that every proposition follows from every other, or that every proposition is provable from no assumptions whatever. Boghossian’s aim was to show that a given inference rule (here tonk-Introduction) could be used to show that the same rule is valid (truth-preserving). But if the conclusion that states the validity of tonk-Introduction is really Boghossian’s statement $P \rightarrow (P t Q)$ above, then one would be justified in asking why the Tonk Argument had to be so roundabout.²⁰ A better argument, because more direct, would have been

$$\frac{\frac{\frac{\overline{P}^{(1)}}{P t Q}^{(t)}}{P \rightarrow (P t Q)}^{(1)}}$$

²⁰ It is *the Tonk Argument* that I am saying is roundabout. I am not saying that Boghossian’s philosophical argument marshalling considerations about the Tonk Argument is roundabout! (I am all too aware that with a paper of this length, such a misguided complaint would render me vulnerable to a *tu quoque*.)

which avoids all truth-predications. It uses the rule of tonk-Introduction, as does the Tonk Argument above.

Perhaps, then, the conclusion that Boghossian ought to have sought to prove should involve the truth predicate, and should more obviously state that the tonk-rules are truth-preserving. Such a conclusion might be

$$T(P) \rightarrow T(P \text{ t } Q).$$

I have already noted the outright provability of any proposition by means of the tonk-rules. Thus one could claim at this point to be able to prove this last conclusion. Here is a proof:

$$\frac{\frac{\frac{\overline{A}^{(1)}}{A}^{(1)}}{A \rightarrow A}^{(1)}}{(A \rightarrow A) \text{ t } (T(P) \rightarrow T(P \text{ t } Q))}^{(tI)}}{T(P) \rightarrow T(P \text{ t } Q)}^{(tE)}$$

But of course anyone presented with this proof would be justified in rejecting it as a reason for accepting its conclusion. For it is patently of the form

$$\frac{\frac{\frac{\overline{A}^{(1)}}{A}^{(1)}}{A \rightarrow A}^{(1)}}{(A \rightarrow A) \text{ tonk } B}^{(tI)}}{B}^{(tE)}$$

that I remarked on above. This is an all-purpose template for the outright proof of any proposition B whatsoever. Why, then, should we give it any credence when the particular substituent for B is $T(P) \rightarrow T(P \text{ t } Q)$?

Perhaps the preceding proof is unsatisfactory for Boghossian's purposes. Moreover, it does, after all, use the elimination rule for tonk as well as its introduction rule. It might be thought that Boghossian could have recourse to one or other of two more suggested proofs. The first of these proofs is just the middle part of the Tonk Argument:

$$\frac{\frac{T(P)}{T(P) \text{ t } T(Q)}^{(tI)} \quad \text{'Meaning Postulate'}}{T(P \text{ t } Q) \leftrightarrow [T(P) \text{ t } T(Q)]}^{(\leftrightarrow I)}$$

In so far as this proof still uses the Meaning Postulate, however, it will be vulnerable to the objections to be raised against that postulate below.

So perhaps Boghossian should be after a proof of the conclusion

$$T(P) \rightarrow T(P \text{ t } Q)$$

whose only applications of tonk-rules are tonk-introductions. The second suggested proof seeks to answer to this requirement.

$$\frac{\frac{\frac{T(P)^{(1)}}{P}}{P \text{ t } Q}}{T(P \text{ t } Q)}}{T(P) \rightarrow T(P \text{ t } Q)}^{(1)}$$

It uses both Semantic Ascent (for sentences with tonk dominant), and Disquotation.

The trouble with this second suggested proof, however, is that its penultimate step

$$\frac{P \text{ t } Q}{T(P \text{ t } Q)}$$

is invalid. One cannot semantically ascend on sentences with tonk dominant – at least, without yet more funny business. This perhaps surprising result will be established in due course.

But even if the second suggested proof did not suffer from this drawback, it would still be an idle exercise. If the proof were valid – that is, if one could semantically ascend on sentences with tonk dominant – then its purported result would not be that the introduction-rule for Prior’s connective tonk was truth-preserving. Rather, it would show only that the introduction rule for ‘regardless whether’ (which, by contrast with tonk, *does* sustain Semantic Ascent) is truth-preserving:

$$\frac{\frac{\frac{T(P)^{(1)}}{P}}{P \text{ regardless whether } Q}}{T(P \text{ regardless whether } Q)}}{T(P) \rightarrow T(P \text{ regardless whether } Q)}^{(1)}$$

For on its own, as noted earlier, the introduction rule

$$\frac{A}{A @ B}$$

can confer on the connective @ only the sense ‘regardless whether’. Prior’s deviant connective tonk is deviant only because he stipulates in addition that it should obey a certain elimination rule. This elimination rule, however, is not the one that would be ‘read off’ the introduction rule if someone were to

be given the introduction rule in an honest attempt to confer a sense on the connective @. The corresponding honest elimination rule would be

$$\frac{A @ B}{A}$$

This is because the introduction rule tells us that a sincere assertion of $A @ B$ would be warranted if, but only if, one had a warrant for the assertion of A alone. Thus one knows that all one is entitled to infer from $A @ B$ is A . That is why @ must be construed as ‘regardless whether’, when one is given only the introduction rule above. Prior’s elimination rule, however, destroys this sensible harmony. It provides for the inference to B from (A tonk B). So, unless we advert to this fact about tonk, we cannot say that any tonk-elimination-eschewing proof of the truth-preserving nature of the introduction rule would be a proof of the truth-preserving nature of the introduction rule *for tonk*. At best it would be a proof of the truth-preserving nature of the introduction rule for ‘regardless whether’.

VIII. CONCEPTUAL ROLE AND MEANING

Indeed, one could go so far as to say that failure of harmony (between Prior’s introduction and elimination rules) means not just that the word ‘tonk’ (supposedly governed by those rules) fails to have a *sense*; it fails also to fill any *conceptual role* within the system of thought of its would-be user. Boghossian thinks that tonk does have a conceptual role, but merely fails to have a sense (or meaning). He writes (p. 32; my italics)

Prior imagined a connective governed by the following introduction and elimination rules:

$$A/A \text{ tonk } B \quad A \text{ tonk } B/B$$

The specification defines a conceptual role; but what meaning does it determine?

Boghossian offers no explanation for how this could be so. He does not say what minimal conditions have been met in order to guarantee a conceptual role for tonk. Timothy Smiley (in personal correspondence) has ventured the guess that the conceptual role of tonk might be ‘that of being what Prior calls a contonktion-forming sign, perhaps in the minimal sense of his §5’.²¹ But to quote Prior here:

... in the metatheory of a purely symbolic game we can define a contonktion-forming sign, either directly in terms of the design of the symbol ... or in terms of the game’s permitted transformations ... such a game would be rather less interesting to play than noughts and crosses.

²¹ See Prior, ‘Conjunction and Contonktion Revisited’, *Analysis*, 24 (1964), pp. 191–5.

It is difficult to see how such a weak sense of ‘conceptual role’ can answer to what Boghossian was requiring of the notion. Two considerations underscore this point.

First, for Boghossian there was still some sort of *inferential activity* involved, with reference to which tonk was supposed to acquire a conceptual role (though not a meaning). Such inferential activity is not a ‘purely symbolic game’, since it is at the very least aiming to transmit, even if only hypothetically, truth or warrant for assertions. The usual notion of conceptual role is that it is defined by reference to the function of a symbol within a system serving to mediate at the very least between perception and action. By comparison with this richer notion, the role of a symbol within a ‘purely symbolic game’ looks rather anaemic.

Secondly, Boghossian does not explain why the conditions for possession of meaning should be more exigent than those for possession of a conceptual role. Conceptual role – the role played, if you like, by an expression within a language of thought – could only correspond, in the case of a logical operator, to the sense or meaning that would attach to its verbal expression in a public language, by virtue of sustainable inferential norms.

How could tonk be faring all right in the language of thought, but only coming unstuck, as it were, when thought goes public? What exactly can the thinker be thinking when deploying tonk with this supposed ‘conceptual role’ in his cognitive system?

IX. BACK TO THE TONK ARGUMENT

The problem of rule-circularity is this: in order to show that a particular rule is truth-preserving, one has to resort to the same rule. It might be thought that this problem would be all the more pronounced if not only that rule, but yet further ones (involving the same connective), were called for in the reasoning. But the foregoing considerations arguably show that this is not so. There is *less* of a problem of rule-circularity, not more of a problem, when one shows that the introduction rule for the conditional is truth-preserving by using not only that introduction rule but also the corresponding elimination rule (namely, *modus ponens*). For it is precisely because *modus ponens* is the properly corresponding elimination rule that we know that the symbol we are using (\rightarrow) really does stand for a proper logical operator (namely, the conditional).

I turn now to point (4) in §II above (p. 627), which focused on the ‘Meaning Postulate’ suggested (but not endorsed) by Wright, and accepted by Boghossian in order to generate his more general sceptical worry:

4. The Tonk Argument is intended to be *a priori*. In particular, the ‘Meaning Postulate’ $T(P \text{ t } Q) \leftrightarrow [T(P) \text{ t } T(Q)]$ (at least, in the right–left direction) is supposed to be *a priori*. The Tonk Argument treats it as an axiom on which the conclusion $P \rightarrow (P \text{ t } Q)$ can be made to rest.

The biconditional

$$T(P \text{ t } Q) \leftrightarrow [T(P) \text{ t } T(Q)]$$

needs to be unproblematically *a priori* if the Tonk Argument is to support a worrying scepticism about logical rule-circularity. But we may ask what the *a priori* warrant is for the inference

$$\frac{T(P) \text{ t } T(Q)}{T(P \text{ t } Q)}$$

Is not the user of this inference rule simply assuming, without the necessary proof, that tonk is what I have called ‘truth-compounding’ (as opposed to ‘truth-distributing’, which would characterize the converse inference)?

In the case of an arbitrary binary connective @, the definitional clause

$$T(A @ B) \leftrightarrow [T(A) @ T(B)]$$

might seem to be clearly in order; for it is, after all, registering the compositionality of the connective @. But that @ is indeed meaningful cannot be achieved by mere stipulation of what look formally like introduction and elimination rules. More is required before meaningfulness is secured, namely, that the stipulated introduction and elimination rules must be in harmony.

This point becomes evident when we try (unsuccessfully) to give, in the case of tonk, proofs *in normal form* of Semantic Ascent, Disquotation, Distribution and Compounding analogous to the proofs which I have provided above for both conjunction and the conditional. Why, the reader may ask, should we insist that such proofs be in normal form? The answer is, for the simple reason that it should be possible to cast *any* proof into normal form! Normal form encodes the most ‘direct’ form of presentation of the line of reasoning in question. As stressed above, a proof in normal form establishes the desired logical connection between its premises and its conclusion by using *only such logical structure as is ‘in’ the premises and/or the conclusion*. It reveals how *ingredient* senses suffice to establish this logical connection.

Suppose, then, that we adopt the obvious truth-predicational variants of the introduction and elimination rules for tonk:

Rules for the truth predicate in relation to tonk:

$$\begin{array}{cc} (Tt\text{-Intro}) & (Tt\text{-Elim}) \\ \frac{T(A)}{T(A \ t B)} & \frac{T(A \ t B)}{T(B)} \end{array}$$

The obvious way to try to prove Distribution of Truth over tonk by means of a normal proof comes unstuck, because the final attempted inference would not be an application of the introduction rule for tonk:

$$\frac{\frac{T(A \ t B)}{T(B)}}{T(A) \ t \ T(B)} \text{ ?? not } t\text{-Intro}$$

Likewise, the obvious way to try to prove Compounding of Truth over tonk by means of a normal proof comes unstuck because the initial attempted inference would not be an application of the elimination rule for tonk:

$$\frac{\frac{T(A) \ t \ T(B)}{T(A)}}{T(A \ t B)} \text{ ?? not } t\text{-Elim}$$

Similarly, the following variations on these attempts come unstuck, for the reasons given:

$$\frac{\frac{T(A \ t B)}{T(A)}}{T(A) \ t \ T(B)} \text{ ?? not } Tt\text{-Elim}$$

$$\frac{T(A) \ t \ T(B)}{\frac{T(B)}{T(A \ t B)}} \text{ ?? not } Tt\text{-Intro}$$

There do not appear to be any other proof-strategies that would work – unless one were to abandon the requirement that the sought proofs must be in normal form. For then Distribution of Truth over tonk could be proved as follows:

$$\frac{\frac{\frac{T(A \ t B)}{T(B)}}{T(B) \ t \ T(A)} \ t\text{-Intro}}{T(A)} \ t\text{-Elim}}{T(A) \ t \ T(B)}$$

with a maximal occurrence of $T(B) \text{ t } T(A)$ standing as conclusion of $t\text{-Intro}$ and as the major premise of $t\text{-Elim}$. Likewise with Compounding of Truth over tonk:

$$\frac{\frac{\frac{T(A) \text{ t } T(B)}{T(B)} \text{ t-Intro}}{T(B) \text{ t } T(A)} \text{ t-Elim}}{T(A)}}{T(A \text{ t } B)}$$

But this is the resort of the desperate. If one is willing to use an irreducibly abnormal proof when deducing Q from P , one might as well use the general template

$$\frac{\frac{P}{P \text{ t } Q}}{Q}$$

to get from the overall premise to the overall conclusion in two steps, rather than applying this template ‘locally’ within the proof, as in the last two examples.

It appears, then, that the Tonk Argument is further wanting. For it rests on the premise called the Meaning Postulate for tonk. This premise asserts both Distribution and Compounding of Truth over tonk. Far from being true and *a priori*, this premise is either (*a priori*) false, or devoid of content. But we have also seen that, ironically, in a properly direct proof of the sought conclusion, this false or contentless premise can be eschewed anyway.

Finally, I note that not even Disquotation and Semantic Ascent can be shown to hold for sentences with tonk dominant. The only possibilities of normal proofs for Disquotation both come unstuck:

$$\frac{\frac{\frac{T(A \text{ t } B)}{T(A)} \text{ ?? not } T\text{-Elim}}{A}}{A \text{ t } B} \qquad \frac{\frac{\frac{T(A \text{ t } B)}{T(B)}}{B} \text{ ?? not } t\text{-Intro}}{A \text{ t } B}$$

Likewise, the only possibilities of normal proofs for Semantic Ascent both come unstuck:

$$\frac{\frac{\frac{A \text{ t } B}{A} \text{ ?? not } t\text{-Elim}}{T(A)}}{T(A \text{ t } B)} \qquad \frac{\frac{\frac{A \text{ t } B}{B}}{T(B)} \text{ ?? not } T\text{-Intro}}{T(A \text{ t } B)}$$

So Tarski's *T*-Scheme fails in both directions for sentences with tonk dominant. The deflationist should take note. Postulated as axioms (for a theory of truth), all instances of the *T*-Scheme will require particular attention to the problem of which sentences (with which logical operators dominant) should be allowed to count as potential substituends in the *T*-Scheme. Tarski's own non-deflationary way was to derive instances of the *T*-Scheme as theorems in his theory of truth. We have just seen that there is no way to do this for sentences with tonk dominant.

This means that the step from line (5) to line (6) in the Tonk Argument is fallacious.

Resolute tonkers will insist that proofs of Semantic Ascent, Disquotation, Distribution and Compounding can be carried through for tonk-involving sentences. But the proofs they are willing (indeed, obliged by their own tonk-rules) to accept are open to a fatal trio of objections:

1. The proofs are irreducibly abnormal
2. They keep ultimate bad company
3. They make no use of the logical structure of their premises and conclusions.

In summary, I have established the following facts about the Tonk Argument.

1. The sole axiom of the Tonk Argument, the so-called Meaning Postulate, is either false or devoid of content (if 'tonk' means Prior's tonk)
2. One of the Tonk Argument's five steps is invalid (if 'tonk' means Prior's tonk)
3. There is an obvious two-step argument for its conclusion, not using the Meaning Postulate
4. That conclusion does not serve Boghossian's philosophical purpose
5. Neither the five-step fallacious argument, nor the correct two-step argument, is about Prior's tonk, except in name. These arguments are really about 'regardless whether'.

Later in his paper (at p. 32), Boghossian correctly claims that tonk has no meaning, for the reason that

... we can readily see that there can be no consistent assignment of truth-value to sentences of the form '*A* tonk *B*' given the introduction and elimination rules for 'tonk'.

He does not remark, however, that this entails that the Meaning Postulate which he had accepted earlier would be either false or devoid of content. He therefore ends, it appears, by undermining the sole reason he had offered to the reader for taking the problem of rule-circularity seriously.

X. FINAL REMARKS

Dummett and Prawitz pioneered the doctrine, now widely held within the community of proof-theorists and philosophers of logic, that harmony between introduction and elimination rules is essential if the operator involved is to have a sense. Disagreements tend to concern how best to explicate harmony, and whether, when one does so, intuitionistic logic is more harmonious than classical logic. But these disagreements have nothing to do with the lessons of harmony in relation to tonk. Tonk is repugnant to intuitionistic and classical logicians alike.

Boghossian's suggested solution to his problem is that the introduction and elimination rules for a (well behaved) logical connective are meaning-constituting. They are therefore not in need of any justification – or at least are none the poorer for having only rule-circular justifications. This is a view already widely held within the above-mentioned community. So I am sure that I shall not be the only proof-theoretically inclined philosopher of logic who would regard Boghossian as having confirmed us in our views.²²

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²² I am grateful to Timothy Smiley for particularly exacting criticisms of an earlier draft. I would also like to thank Nuel Belnap, John Burgess, Michael Hand, Alan Millar and Joseph Salerno for helpful correspondence, and Paul Boghossian for clarificatory conversation. The submitted version was improved by the acute remarks of two anonymous referees.