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## ARTICLE

# MIND, MATHEMATICS AND THE *IGNORABIMUSSTREIT*<sup>1</sup>

Neil Tennant

### INTRODUCTION

Certain developments in recent philosophy of mind that contemporary philosophers would regard as both novel and important were fully anticipated by writers in (or reacting to) the tradition of *Naturphilosophie* in Germany in the mid-to-late-1800s, as well as in the works of other writers during that period in Britain and France. The ideas or theses in question are: materialism, both reductive and non-reductive; supervenience and mysterianism; the explanatory gap; the knowledge argument; and functionalism.

Making this claim plausible is the main historical task of this paper. In the course of doing so, some conceptual issues are addressed. We shall encounter an interesting intersection of different lines of thought in the so-called *Ignorabimusstreit* concerning what we can know about consciousness, the nature of physical existence, and mathematical truth. The nineteenth-century *Naturphilosophen* and their heirs in the early twentieth century grappled with epistemological problems in the broadest of settings. Their work is directly relevant to the modern debate between realists and anti-realists, in so far as the latter debate turns on the question of what, in principle, is knowable.

<sup>1</sup>For encouragement or helpful comments thanks are owed to José Luis Bermúdez, David Chalmers, Patricia Churchland, Paul Churchland, Mario de Caro, Daniel Dennett, Gabriel Finkelstein, Frank Jackson, Richard Garner, Stephen Gaukroger, Ken Gemes, Samuel Guttenplan, Glenn Hartz, Jim Hopkins, Vittorio Hösle, Nicholas Jones, David Joyce, Robert Kirk, William Lycan, Peter Milne, Andrew Oldenquist, Paolo Parrini, Volker Peckhaus, Philip Pettit, Adam Podlaskowski, Diana Raffman, Nicholas Rescher, Georges Rey, David Rosenthal, George McDonald Ross, Tamar Rudavsky, Florian von Schilcher, Wilfried Sieg, Allan Silverman, Jack Smart, Richard Sorabji, David Spurrett and Michael Tye. The author is responsible for all errors that remain. Unless otherwise indicated in the text, translations from German are by the author. The original German passages will be confined to the endnotes wherever possible.

It is remarkable how quickly the contributions of the *Naturphilosophen* seem to have faded in collective philosophical memory, at least in the English-speaking world, to be acknowledged no more. Yet contemporary English-speaking philosophers stand in a philosophical tradition significantly influenced by Quine, who gave arresting expression to a scientifically minded philosophy due in large measure to Carnap.<sup>2</sup> Carnap, in turn, in *Der Logische Aufbau der Welt* (1928), gave ample signposting into the intense discussion of the ‘psychophysical problem’ by many of his German precursors in the nineteenth century. Neither Quine nor Carnap – and a fortiori, none of their twentieth-century contemporaries – is the focus of this historical and conceptual study. Quine and Carnap are mentioned simply because they are two of the giants of twentieth-century philosophy whose works point the reader straight into the more distant historical period – the second half of the nineteenth century – which is the focus of this study. If one follows the bibliographic leads from Quine via Carnap, one finds that much of what might be thought original in contemporary philosophy of mind is already in the intellectual fossil record.

A caveat is needed at the outset. This study, as already indicated, is partly historical, and partly conceptual. Its wider aim is to reaffirm the philosophical value of what is almost (in the English-speaking world, at least) a lost tradition of thought. It is no part of this project to trace lines of intellectual influence, or to classify continuities and variations in thought down to the present day. On the contrary: the intention is to demonstrate just how clearly much of what is thought to be current was anticipated, in considerable detail, in the great cross-Channel conversations of the second half of the nineteenth century. Nor are any subtle hermeneutic problems involved in establishing this. Note also that there is no need to give a detailed historical account of the views of the so-called British Emergentists in the early twentieth century, such as C. D. Broad. For the British Emergentists are *late* enough not to qualify for the credits to be (re-)assigned here. (C. D. Broad’s *The Mind and Its Place in Nature* (1925) is devoid of any mention of, or engagement with, the views of any of those nineteenth-century German or British thinkers on whom this study will focus.)

Perhaps the hardest problem in contemporary philosophy of mind, after the problem of consciousness itself, is how to find anything new to say about the problem of consciousness. The depth of the problem of consciousness was acknowledged in its designation by German thinkers in the nineteenth century as a *Welträtsel* – a riddle of the universe. It was the renowned physiologist-philosopher-psychologist Emil du Bois-Reymond who gave the classic statement of epistemological pessimism about some of these riddles. He did this in his famous lecture ‘Über die Grenzen des Naturerkennens’, delivered in 1872, and in his follow-up lecture ‘Die sieben Welträtsel’, delivered in 1880, in the wake of the general furore that the

<sup>2</sup>See the present author’s paper (1994).

first of these lectures had created.<sup>3</sup> In due course the polymath developmental biologist and philosopher Ernst Haeckel responded with his famous *fin de siècle* treatise in support of a countervailing optimism (see Haeckel (1899), translated as Haeckel (1900)). These and related works make one realize that contemporary writers are engaged in a re-play, more than a century later, of what du Bois-Reymond called a critical discussion striking all notes ‘from happy praise in agreement, to the most dismissive censure’.<sup>4</sup>

In his 1872 lecture du Bois-Reymond listed seven *Welträtsel*:<sup>5</sup>

1. The nature of matter and force.
2. The origin of motion.
3. The origin of life.
4. The (apparently preordained) orderly arrangement of nature.
5. The origin of simple sensation and consciousness.
6. Rational thought and the origin of the cognate faculty, speech.
7. The question of freedom of the will.

du Bois-Reymond regarded problems (1), (2) and (5) as ‘entirely transcendental and insoluble’. He had no way of knowing what insights (short of complete solutions) for (1) and (2) were subsequently to be afforded by quantum mechanics and relativistic cosmology. He was prescient, however, with (5). For it is (5) that has most obstinately resisted all attempts at its solution, as du Bois-Reymond fully anticipated.

## NINETEENTH-CENTURY ANTICIPATIONS OF VARIOUS CONTEMPORARY POSITIONS

### Philosophical materialism

In light of the fifth problem’s pedigree, it should come as no surprise that some contemporary philosophers venture to describe it as *hard* (see especially David Chalmers (1996)). This has provoked reactions that echo the hardline materialists of the nineteenth century. Today’s eliminative materialism, in what it envisages as likely sufficient descriptive and explanatory resources within a completed science of the mind/brain, is hardly a patch on Richard Avenarius (1876). In his remarkable little tract he put forward the view that our mental processes, from memory and associative thinking through to logical ratiocination, could all be explained in terms of the physicists’ principle of least action. Avenarius was therefore perhaps the first explicitly *reductive* materialist in claiming the systematic

<sup>3</sup>All page references to the 1880 lecture will be to its reprinting in du Bois-Reymond (1898).

<sup>4</sup>*vom freudig zustimmenden Lobe bis zum wegwerfendsten Tadel*’.

<sup>5</sup>The list, in English translation, is taken from Haeckel (1900, p. 15).

replaceability of mentalistic descriptions and explanations with physicalistic ones. (By contrast, du Bois-Reymond would have to be classified as a non-reductive materialist, given the absence from his writings of any explicit or implied claim of such replaceability.)<sup>6</sup>

Modern 'eliminative materialists' such as Patricia and Paul Churchland (1981, 1986), and even a 'biological essentialist' such as John Searle (1992), follow paths whose beginnings trace back at least to the so-called 'vulgar materialists' in the first half of the nineteenth century in Germany. It is not being suggested here that the important and distinguishing parts of the later contributions were among those anticipated in the nineteenth century. In particular, what the Churchlands introduced was the idea that folk psychology is a *false* theory that should be replaced by a scientific theory – it is not an indispensable theory serving as an adequacy constraint on its scientific extensions; and what Searle introduced was the idea that there is something essential about the actual biological substrate (our brains and nervous systems) in making consciousness possible. Those two main differentiae having been noted, however, it is still worth remarking that the core, scientifically inspired *materialism* involved is of impressive vintage.

The vulgar materialists in the nineteenth century included, besides Avenarius, such figures as Carl Vogt, who claimed:<sup>7</sup>

that all those capacities that we grasp under the designation of mental activities are only functions of the brain, or, to put it somewhat crudely, that thoughts stand in much the same relation to the brain as gall does to the liver, or urine to the kidneys.<sup>8</sup>

Another such figure was Rudolf Virchow, who '... pointed out triumphantly that, although as an anatomist he had already dissected so-and-so many bodies, he had not been able to discover a soul in any one of them'.<sup>9</sup>

<sup>6</sup>Thanks to Nicholas Jones for raising the need to clarify whether the materialism at issue would have been reductive or non-reductive.

<sup>7</sup>See Vogt (1855, p. 32); what du Bois-Reymond calls his 'cheeky claim' [*kecke Behauptung*] is quoted in du Bois-Reymond (1898, p. 49).

<sup>8</sup>'daß alle jene Fähigkeiten, die wir unter den Namen Seelenthätigkeiten begreifen, nur Funktionen des Gehirns sind, oder, um es einigermaßen grob auszudrücken, daß die Gedanken etwa in demselben Verhältnisse zum Gehirn stehen, wie die Galle zu der Leber oder der Urin zu den Nieren.'

<sup>9</sup>'... verwies triumphierend darauf, daß er, obwohl er als Anatom schon soundsoviele Körper sezirt habe, in keinem einzigen eine Seele habe entdecken können'. This quote is taken from Udo Leuschner; see <http://www.udo-leuschner.de/psychologie/psych5.htm>. Now of course the quote could go either way – since it is also compatible with Cartesian dualism. (The author is indebted here to Diana Raffman.) Virchow, however, was no dualist, and did not intend his remark to support dualism.

**Mysterianism and supervenience . . .**

This vulgar materialism needed a gentle corrective.

The corrective came from within the ranks of naturalizing scientists. In the first edition of his classic work *Lessons in Elementary Physiology*, in 1866, Thomas H. Huxley flatly confessed:

How it is that anything so remarkable as a state of consciousness comes about as a result of irritating nerve tissue, is just as unaccountable as the appearance of the Djinn when Aladdin rubbed his lamp.<sup>10</sup>

Huxley thus notes the ‘unaccountability’; while a contemporary mysterian such as Colin McGinn (1989, 1999) seeks to go further by offering a radically realist account of the unaccountability itself. We are, says McGinn, *cognitively closed* with respect to the supposed natural property of the brain that would explain consciousness (1989, pp. 350 & 352):

we are cut off by our very cognitive constitution from achieving a conception of that natural property of the brain (or of consciousness) that accounts for the psychophysical link . . . the felt mystery comes from our own cognitive limitations, not from any objective eeriness in the world . . . We should . . . be alert to the possibility that a problem that strikes us as deeply intractable, as utterly baffling, may arise from an area of cognitive closure in our ways of representing the world.

McGinn proceeds to argue that ‘no coherent method of concept introduction will ever lead us to’ the sought natural, explanatory property. It lies beyond our perceptual concepts (which are ‘geared to representing a spatial world’); and it lies moreover beyond the scope of the best explanations that we might infer from physical data – largely because ‘a certain principle of *homogeneity* operates in our introduction of theoretical concepts on the basis of observation’ (358).

McGinn adds the wrinkle that we human beings are stumped by the problem because of the deficiencies of our own cognitive apparatus; and leaves open (but only just) the possibility that *other* possible minds (which is to say, even if not explicitly: possible *embodied* minds) might succeed where we are doomed to fail. If this is so, however, then we human beings are in principle debarred from ever knowing it to be the case; and an anti-realist would hardly be heartened by this mitigating feature of McGinn’s

<sup>10</sup>Cited by Güven Güzeldere (see 1997, p. 47, n. 6. The citations by Colin McGinn (1989, p. 349) and Michael Tye (1995, p. 15) begin with ‘How’, rather than ‘[H]ow’. McGinn moreover has ‘where’ in place of ‘when’; uses ‘initiating’ rather than ‘irritating’; and attributes the quote to Julian Huxley, rather than T.H. Huxley.

mysterianism. Indeed, McGinn comes very close to dismissing the possibility of merely *relative* cognitive closure, when he writes:

if we suppose that *all* concept formation is tied to perception and introspection, however loosely, then *no* mind will be capable of understanding how it relates to its own body – the insolubility will be absolute.

(loc. cit. 361)

There being no alternative direction for one's mental gaze that one can readily think of, besides *outwards* (perception) or *inwards* (introspection), one might be inclined to apply *modus ponens* here on McGinn's behalf – and conclude that cognitive closure is absolute.

Michael Tye (2000, p. 23 and 29, n1), writes that McGinn's view, which Tye calls *deeply pessimistic physicalism*, 'seems to trace back to Du Bois-Reymond 1885–87'. Indeed, it traces back even further, to Huxley. His *Lessons*, first published in 1866, contained both the thesis of supervenience and the mysterian thesis. At p. 188 of the third edition of 1872, Huxley expresses the mysterian thesis as follows:

We class sensation along with *emotions*, and *volitions*, and *thoughts*, under the common head of *states of consciousness*. But what consciousness is, we know not; and how it is that anything so remarkable as a state of consciousness comes about as the result of irritating nervous tissue, is just as unaccountable as any other ultimate fact of nature.<sup>11</sup>

<sup>11</sup>The original quote from p. 193 of the first edition was 'But what consciousness is, we know not; and how it is that anything so remarkable as a state of consciousness comes about as the result of irritating nervous tissue, is just as unaccountable as the appearance of the Djinn when Aladdin rubbed his lamp in the story, or as any other ultimate fact of nature.'

In the second edition of 1868, p. 210, Huxley deleted the words 'in the story': 'But what consciousness is, we know not; and how it is that anything so remarkable as a state of consciousness comes about as the result of irritating nervous tissue, is just as unaccountable as the appearance of the Djinn when Aladdin rubbed his lamp, or as any other ultimate fact of nature.'

Finally, in the third edition of 1872, p. 188, the passage becomes Djinn-less: 'But what consciousness is, we know not; and how it is that anything so remarkable as a state of consciousness comes about as the result of irritating nervous tissue, is just as unaccountable as any other ultimate fact of nature.'

In the scholarly sleuthing involved in discovering these changes, invaluable assistance was rendered by Judith Harrison, Head of the Humanities Reference Services at the British Library and by her assistant Erin Caseley; by Reed Lowrie, Science Reference Librarian and Biomedical Specialist at the John Crerar Library of the University of Chicago; by Ingrid Radkey, Librarian at the Marian Koshland Bioscience and Natural Resources Library, UC Berkeley; and by Jette Nielsen, Assistant Librarian at the Wellcome Library for the History and Understanding of Medicine in London. It is difficult to determine the exact reasons for Huxley's changes to his text. It is reasonable, however, to surmise that, even though he never intended his metaphor about Aladdin and the Djinn to be misunderstood in this way, it might have come to Huxley's attention that some readers had gained either the mistaken impression that he was inclined to take seriously the possibility of supernatural phenomena such as Djins – especially after his

Later, at pp. 300–1, Huxley expresses the supervenience thesis along with an even stronger version of the mysterian thesis as follows:

There is no doubt that a molecular change in some part of the cerebral substance is an indispensable antecedent to every phenomenon of consciousness. And it is possible that the progress of investigation may enable us to map out the brain according to the psychical relations of its different parts. But supposing we get so far as to be able to prove that the irritation of a particular fragment of cerebral substance gives rise to a particular state of consciousness, *the reason of the connection between the molecular disturbance and the psychical phenomenon appears to be out of the reach, not only of our means of investigation, but even of our powers of conception.* [Emphasis added.]

The hypothesis of cognitive closure, according to McGinn (1999, p. xii), came to him in ‘a sudden revelation one dark night in Oxford in 1988...’.

As the last line of the foregoing quote from Huxley reveals, the hypothesis had already been advanced in a classic Victorian textbook that saw six different editions and at least as many reprints, and with whose engaging image of Aladdin and the Djinn McGinn began his own subsequent meditations (1989).

McGinn did not source this oft-quoted claim of Huxley, so it is difficult to tell whether he took it from Huxley’s *Lessons*. A hundred pages beyond the mention of Aladdin and the Djinn is Huxley’s clear statement of cognitive closure that has just been quoted. We shall see presently that it is also unlikely that McGinn was aware of du Bois-Reymond’s celebrated work, in which we find an extended argument, by reference to a thought-experimental Laplacean demon, for absolute cognitive closure. One might venture the qualification ‘absolute’ here, since the Laplacean demon is supposed to represent the ultimate knower, no matter whether it be disembodied, and no matter what – if embodied – its organic constitution might be.

In imputing a supervenience thesis to Huxley (and to John Tyndall – see below), one is not suggesting that they formulated supervenience as the kind of delicately modalized claim preferred by some contemporary analytical philosophers of mind (as, for example, by Jaegwon Kim (1987)). The modalizing can be regarded as a variational extra; the main breakthrough, before that variation can be undertaken, consists simply in the conception of the mental being *actually determined* by the physical. This determination needs only to be construable as *contemporaneous* and *not*

deletion of the words ‘in the story’; or (as has been suggested in private correspondence by David Joyce) the mistaken impression that Huxley ‘supported some divine intervention in the creation of consciousness’.

*necessarily causal*.<sup>12</sup> That allows for its later refinement into the idea that in fact the kind of level-to-level determination that is involved in supervenience is both *metaphysically necessary* and *necessarily non-causal*. Huxley and Tyndall at least deploy a notion of supervenience recognizably explicable as the thesis of physical determination propounded by Hellman and Thompson (1975). (The main difference would be over whether to include *physiological* facts in the supervenience basis, or whether to restrict this basis so that it contains only (micro-) *physical* facts.) This same thesis of physical determination was the supervenience thesis at issue when George Bealer (1978) subsequently attempted to show that supervenience would entail reductionism, and when the present author (1985) argued, *contra* Bealer, that it would not.

### ... and the explanatory gap

Huxley's views no doubt affected the *Zeitgeist* of his scientific contemporaries. On 19 August 1868 Tyndall gave an address to a meeting of the Mathematical and Physical Section of the British Association in Norwich. The title was 'Scope and limit of scientific materialism' (1874, pp. 107–22). It is a remarkable document, one from which one cannot forebear quoting at some length. Contemporary philosophical commentary will be quite uncalled for. The reader will easily see Tyndall's clear expressions of the same views on supervenience and mysterianism as we saw in Huxley – along with the clearest possible expression of the so-called *explanatory gap*. This useful phrase, due to Joseph Levine (1983), is of much more recent provenance than the actual problem of the explanatory gap, which, as we have seen, goes back at least to T. H. Huxley in 1866.

in the eye of science *the animal body* is just as much the product of molecular force as the stalk and ear of corn, or as the crystal of salt or sugar . . . Our difficulty is not with the *quality* of the problem, but with its *complexity*; and this difficulty might be met by the simple expansion of the faculties which we now possess . . . Associated with this wonderful mechanism of the animal body we have phenomena no less certain than those of physics, but between which and the mechanism we discern no necessary connection. A man, for example, can say, *I feel, I think, I love*; but how does *consciousness* infuse itself into the problem? The human brain is said to be the organ of thought and feeling . . . I hardly imagine there exists a profound scientific thinker, who has reflected upon the subject, unwilling to admit the extreme probability of the hypothesis that, for every fact of consciousness, whether in the domain of sense, of thought, or of emotion, a definite molecular condition of motion or structure is set up in the brain; or who would be disposed even to deny that if the motion

<sup>12</sup>One notable exception is McGinn, who claims (loc. cit. 353) 'Brain states *cause* conscious states . . .'. (Emphasis added.)

or structure be induced by internal causes instead of external, the effect on consciousness will be the same?...

The relation of physics to consciousness being thus invariable, it follows that, given the state of the brain, the corresponding thought or feeling might be inferred; or given the thought or feeling, the corresponding state of the brain might be inferred.

Note that this last claim is even stronger than a claim of mere supervenience; and indicates something more like a type–type identity thesis. Tyndall continues:

But how inferred? It would be at bottom not a case of logical inference at all, but of empirical association. You may reply that many of the inferences of science are of this character; the inference, for example, that an electric current of a given direction will deflect a magnetic needle in a definite way; but the cases differ in this, that the passage from the current to the needle, if not demonstrable, is thinkable, and that we entertain no doubt as to the final mechanical solution of the problem. *But the passage from the physics of the brain to the corresponding facts of consciousness is unthinkable.* [Emphasis added.] Granted that a definite thought, and a definite molecular action in the brain occur simultaneously; *we do not possess the intellectual organ, nor apparently any rudiment of the organ, which would enable us to pass, by a process of reasoning, from the one to the other.* [Emphasis added.] They appear together, but we do not know why. Were our minds and senses so expanded, strengthened, and illuminated as to enable us to see and feel the very molecules of the brain; were we capable of following all their motions, all their groupings, all their electric discharges, if such there be; and were we intimately acquainted with the corresponding states of thought and feeling, we should be as far as ever from the solution of the problem, ‘How are these physical processes connected with the facts of consciousness?’ *The CHASM between the two classes of phenomena would still remain intellectually impassable.* [Both emphases added.]...

In affirming that the growth of the body is mechanical, and that thought, as exercised by us, has its correlative in the physics of the brain, I think the position of the ‘Materialist’ is stated, as far as that position is a tenable one. I think the materialist will be able finally to maintain this position against all attacks; but I do not think, in the present condition of the human mind, that he can pass beyond this position. I do not think he is entitled to say that his molecular groupings and his molecular motions *explain* every thing. In reality, they explain nothing. The utmost he can affirm is the association of two classes of phenomena, of whose real bond of union he is in absolute ignorance. *The problem of the connection of body and soul is as insoluble in its modern form as it was in the prescientific ages.* [Emphasis added.]<sup>13</sup>

<sup>13</sup>Güven Güzeldere quotes just the closing words ‘... as insoluble in its modern form as it was in the prescientific ages’ (1997, p. 47).

This is a beautifully lucid expression of the explanatory gap. The only difference is that the word ‘gap’ has not been used. Instead, Tyndall uses the much more emphatic word ‘chasm’. This is arguably much better than ‘gap’, whose connotation is almost always of a relatively *small* separation, and one liable to be bridged or closed.

## EMIL DU BOIS-REYMOND

### The explanatory gap, continued

Tye (1999) writes ‘A referee for *Mind* notes that the first philosopher to have used the term ‘gap’ in connection with consciousness was [d]u Bois-Reymond (1885–7)’. And in his later book Tye (2000) incorporating this article as a chapter, at p. 39, n1, the claim becomes ‘The first philosopher to have used the term “gap” in connection with consciousness (to my knowledge) was [d]u Bois-Reymond’. Be that as it may, (some of) the credit surely goes to Huxley and Tyndall for articulating the thought that would nowadays be expressed by saying that there is an explanatory gap in the materialist’s account of mind and its place in nature. In this connection it is worth pointing out that Joseph McCabe, the translator of Haeckel’s *Die Welträtsel* referred, in his Translator’s Preface, to Huxley and Tyndall as Haeckel’s great precursors:<sup>14</sup>

there is none to read aright for us, in historic retrospect, what after ages will probably regard as the most salient feature of the nineteenth century – the conflict of theology with philosophy and science. The pens of our Huxleys, and Tyndalls, and Darwins lie where they fall; there is none left in strength among us to sum up the issues of that struggle with knowledge and sympathy.

The term that du Bois-Reymond used was ‘*Kluft*’:

... unser Naturerkennen gelangt an eine Kluft, über die kein Steg, kein Fittig trägt: wir stehen an der anderen Grenze unseres Witzes.

[... our knowledge of nature reaches a chasm, over which no bridge, no wing will carry us; we stand at the other limit of our wits.]

(1898, p. 33)

<sup>14</sup>See Haeckel (1900, p. xi). McCabe’s translation was first published in 1900. It was this work of Haeckel which gave us the aphorism ‘Ontogeny recapitulates phylogeny’, upon which James Grier Miller played his now famous variation, as quoted by Quine in the front matter of *Word and Object*: ‘Ontology recapitulates philology’.

It was Peter Bieri who translated the German word '*Kluft*' in this passage as the less emphatic English word 'gap'.<sup>15</sup> Bieri might have done better by following G. E. M. Anscombe's celebrated translation of Wittgenstein's *Philosophical Investigations*, where at §412 the original German reads:<sup>16</sup>

Das Gefühl der Unüberbrückbarkeit der Kluft zwischen Bewußtsein und Gehirnvorgang: Wie kommt es, daß das in die Betrachtungen des gewöhnlichen Lebens nicht hineinspielt?

In Anscombe's translation this is rendered as:

The feeling of an unbridgeable *gulf* between consciousness and brain-process: how does it come about that this does not come into the considerations of our ordinary life? [Emphasis added.]

Although, in the German, it is the *unbridgeability* of the gulf that is the focus of the feeling, rather than the gulf itself, Anscombe nevertheless has the right choice of term in *gulf*. It is with *gulf*, and the cognate English word *chasm*, that the German word *Kluft* keeps connotative company. The English word *gap*, here, just does not get it right.

Finally, du Bois-Reymond was quick to point out, also, that his riddle concerned consciousness as a special case; his explanatory *Kluft* was not a matter of pessimism over the prospects for lower-level explanations of other emergent *physical* phenomena, such as the phenomenon of life. Emergent properties and processes at these lower levels *did* admit of purely physical explanation. By contrast, the emergence of consciousness presents a wholly new impossibility *in principle*. In order to establish this, du Bois Reymond employs a notion of so-called 'astronomical' knowledge (1898, p. 75):

I called astronomical knowledge of a material system such knowledge as we would have of a planetary system, if all observations were correct, and all theoretical difficulties fully overcome.<sup>17</sup>

He then claims that astronomical knowledge would encompass the origin of life (1898, p. 76):

The first origin of life in itself has nothing to do with consciousness. What is at issue here is only the ordering of atoms and molecules, an introduction of certain conditions. Consequently it is not only thinkable in this case that we

<sup>15</sup>See Bieri (1995, p. 45), a paper displaying a rare awareness of how distant is the provenance of much of what might be thought of as 'contemporary' philosophy of mind.

<sup>16</sup>Thanks to Jim Hopkins for pointing this out.

<sup>17</sup>'Ich nannte astronomische Kenntnis eines materiellen Systemes solche Kenntnis, wie wir sie vom Planetensystem hätten, wenn alle Beobachtungen unbedingt richtig, alle Schwierigkeiten der Theorie völlig besiegt wären.'

could have astronomical knowledge, but this astronomical knowledge would also satisfy our need for causal knowledge in connection with the first origin of life, as much as it would in connection with the movements of heavenly bodies.<sup>18</sup>

Du Bois-Reymond's conception of the limits, in principle, to our possible knowledge of nature is emphatically at odds with McGinn's (by contrast, very casual-seeming) assertion that '[c]onsciousness is like life in th[e] respect' that it is not a 'primitive brute fact', that it did not arise 'by some form of miraculous emergence', but rather that 'there must be some natural account of how [each] comes from matter, whether or not we can know it'. (1989, p. 353) McGinn does not address du Bois-Reymond's celebrated argument, even though concurring with the latter's famous thesis about the limits of knowledge when he (McGinn) writes 'we should look seriously at the idea that the mind-body problem brings us bang up against the limits of our capacity to understand the world' (loc. cit. 354). Du Bois-Reymond had already taken this idea, and with the utmost seriousness, to the very limits of its expressibility.

He did this as part of his corrective campaign not only against the *Vulgärmaterialisten*, but also against the vitalists. Du Bois-Reymond flew the same banner that had been unfurled by Huxley and Tyndall across the English Channel. In a lecture to the Leibniz section of the Academy of Sciences in 1880, in his capacity as permanent secretary of the Prussian Academy of Sciences, he began by recalling the by then famous lecture he had given eight years earlier to a public meeting of the *Versammlung Deutscher Naturforscher und Ärzte*.<sup>19</sup> He bluntly confessed: (1898, p. 69)

It actually seemed to me a trivial truth that it was impossible, on the one hand, to grasp the nature of matter and force, and, on the other hand, to explain consciousness, even at the lowest level, in mechanical terms.<sup>20</sup>

Clearly du Bois-Reymond was pointing out the existence of what is nowadays called the 'explanatory gap'. Moreover, he insists that reference to it is to be found in even earlier writers. In his 1875 literary tribute

<sup>18</sup>Die erste Entstehung des Lebens hat an sich mit dem Bewußtsein nichts zu schaffen. Es handelt sich dabei nur um Anordnung von Atomen und Molekeln [sic], um Einleitung gewisser Bedingungen. Folglich ist nicht bloß astronomische Kenntnis dessen denkbar, was man Urzeugung . . . nennt, sondern diese astronomische Kenntnis würde auch in bezug auf die erste Entstehung des Lebens unser Kausalitätsbedürfnis ebenso befriedigen, wie in bezug auf die Bewegungen der Himmelskörper.'

<sup>19</sup>For the original lecture given in 1872, see du Bois-Reymond (1998). It was from this lecture that some excerpts are translated by Bieri (1995).

<sup>20</sup>Die Unmöglichkeit, einerseits das Wesen von Materie und Kraft zu begreifen, andererseits das Bewußtsein auch auf niederster Stufe mechanisch zu erklären, erschien mir eigentlich als triviale Wahrheit.'

'La Mettrie', (1912, p. 528), he cites also La Mettrie's *L'Homme machine*, 1865, for the view that 'We shall never grasp the being of what we call matter and force, and we shall never grasp how it is that matter thinks'.<sup>21</sup> Also, du Bois-Reymond (1898, p. 64) cites Tyndall (1874).<sup>22</sup>

Du Bois-Reymond cites various passages from Leibniz, among them the famous thought-experiment that Leibniz had given in §17 of *The Monadology*, to show that '*perception*, and that which depends on it, *cannot be explained mechanically*, that is to say by figures and motions'.<sup>23</sup> §17 of *The Monadology* continues:

Suppose that there were a machine so constructed as to produce thought, feeling, and perception, we could imagine it increased in size while retaining the same proportions, so that one could enter as one might a mill. On going inside we should only see the parts impinging upon one another; we should not see anything which would explain a perception. The explanation of perception must therefore be sought in a simple substance, and not in a compound or in a machine. Moreover, there is nothing else whatever to be found in the simple substance except just this, viz. perceptions and their changes. It is in this alone that all the *internal actions* of simple substances must consist. [All emphases in original.]<sup>24</sup>

In his trenchant reaffirmation of what du Bois-Reymond takes to be Leibniz's line here, the precise nature of his (du Bois-Reymond's) explanatory *Kluft* bears some emphasizing. He stresses the *impossibility* of our ever knowing how consciousness arises from the physical workings of the brain, in so far as these workings are describable in 'mechanical terms'. Yet at the same time he believes that the fundamental laws of physics are not simply empirically well-confirmed hypotheses that are in principle revisable

<sup>21</sup>Nie werden wir . . . das Wesen dessen begreifen, was wir Materie und Kraft nennen und nie werden wir begreifen, wie Materie denkt.'

<sup>22</sup>Du Bois-Reymond calls Tyndall '*mein Freund*'. The Irish-born Tyndall moved to England at the age of 22, and went to Marburg in 1848, at the age of 28, to study for a doctorate, whereupon he returned in 1853 to the position of Professor of Natural Philosophy at the Royal Institution. (See Mandell & Sell (2002) for further details.) The friendship with Tyndall to which du Bois Reymond referred began when they met in Berlin, where Tyndall was studying with Magnus, du Bois-Reymond's teacher. (Here I am indebted to Gabriel Finkelstein.)

<sup>23</sup>Translation taken from the Parkinson edition 1973, p. 181. Emphases in original.

<sup>24</sup>Paul Churchland (1996, pp. 191–2) reminds us of this brief passage, but with some vivid embellishment: '[Leibniz] has us imagine that we are shrunk to the size of the smallest mite, thence to enter into the machinery of the brain as a man might enter a giant mechanical mill, one filled with levers, pulleys, gears, and all of the other intricacies to be contrived within purely physical machinery . . . However carefully we might examine that vast mill's mechanical economy, claimed Leibniz, it is obvious that we would never catch therein the slightest glimpse of a thought, or a desire, or a sensation. Those phenomena, he was thus assured, must belong to a quite different order of reality.'

So we see that Churchland's mite is a mite extra-textual, even if not extraterrestrial.

in the light of recalcitrant evidence. Rather, for du Bois-Reymond, those laws are (1898, p. 16) ‘mathematically representable, and carry *the same apodeictic certainty as the laws of mathematics*’ (emphasis added).<sup>25</sup>

Coming from a writer with (what might be regarded nowadays as) such a ‘naively Kantian’ epistemological view about the fundamental laws of physics, the assertion of an explanatory *Kluft* is all the more arresting. Nor should one think that, with such impossibly high standards for fundamental laws of physics, it is no wonder that du Bois-Reymond believes it would be impossible to explain the emergence of consciousness on the resulting slender basis. On the contrary, du Bois-Reymond clearly did not think the domain of science impoverished; he was simply mistaken in his own conception of the epistemic standards that the then-accepted physical laws could and did satisfy. That is, he was definitely allowing all of the then-‘known’ physics at the ‘lower level’, as it were; but he was misdescribing its logical status in our epistemic scheme. He was definitely not introducing new, overly exigent standards for what was to count as physical knowledge, and then claiming that, on the slender basis of such physical laws as would then survive, it would be impossible to explain the emergence of consciousness.<sup>26</sup>

Moreover, du Bois-Reymond contrasts the impossibility of explaining the mental in terms of the physical with the very real (and subsequently realized) possibility of explaining various ‘higher’ physical levels in terms of some favoured ‘lower’ level. His main example (quoted above) of a physical phenomenon at a higher level is that of the origin of life; and he regards this higher-level phenomenon as fully explicable by the ‘astronomical’ (i.e. logically complete) knowledge of physics that one can imagine oneself, in principle, as capable of attaining. He would no doubt have made the same claim, explicitly, for the phenomena of, say, inorganic chemistry – had he been pressed for further clarification of which phenomena one could, and which phenomena one could not, hope to explain by recourse to presumed ‘astronomical’ knowledge of physics. It is only with the phenomenon of *consciousness* that the explanatory *Kluft*, for du Bois-Reymond, arises. He is a thoroughgoing physicalist with regard to the various natural sciences.

In exhibiting the lower-level physical basis for the emergence of some higher-level property, du Bois-Reymond clearly understands one to be explaining the (relative) *necessitation* of the emergent property by the subvening basis in question. Thus sugar’s solubility in water is explained, hence necessitated, by its crystalline structure; the phenomena of heritability and sexual reproduction are explained, hence necessitated, by the molecular-biological details of the genetic code; and so on. Regarding the latter

<sup>25</sup>... mathematisch darstellbar, und tragen in sich *dieselbe apodiktische Gewissheit, wie die Sätze der Mathematik.*’ (Emphasis added.)

<sup>26</sup>Thanks to Volker Peckhaus for raising the possible reaction against which these remarks are directed.

example, even though the details of the genetic code were not to be discovered until decades after du Bois-Reymond's writings on these matters of emergence, he was uncannily prescient in an age when vitalism was the order of the day (1898, pp. 31–2):

It is . . . a misunderstanding to see, in the first appearance of living creatures on earth or on another planet, something supernatural, something other than an extremely difficult mechanical problem. . . . If we could manufacture the conditions under which living creatures first emerged, the way we can for certain, if not for every, crystal, then in accordance with the Principle of Actualism living creatures would emerge today, just as they did then.<sup>27</sup>

Once we understand the lower-level workings of the world in light of our overall scientific theory, then, we grasp the inevitability, indeed, the necessity, of the emergent phenomena whose micro-constitutions have been laid bare. This is of course no absolute necessity; it is a necessity only relative to the obtaining of the posited lower-level conditions. However, it is exactly the kind of necessity that the contemporary explanatory-gap theorist has in mind when contrasting it (in cases such as solubility and the emergence of life) with the apparent *absence* of any similar kind of necessity when it comes to the emergence of conscious phenomena from their physical substrates. However, du Bois-Reymond is already alive to this contrast as well. On the supposition that one was in possession of the relevant 'astronomical knowledge', he asserts that:

The workings – which are arbitrary and *not necessarily bound up with sensation* – of the central parts [of the brain] . . . the exchange of matter in the brain and spinal cord, and the like, would be exhaustively known. In addition, the material processes that are correlated in time – always, *and so perhaps necessarily* – with mental processes would likewise be rendered completely transparent. (du Bois-Reymond, 1898, p. 39; Emphases added.)<sup>28</sup>

The hapless 'astronomical' savant would not be able to avoid a possibility of predictive and explanatory powerlessness even more dismaying than that of spectral inversion. Speaking of two different physical stimuli – the striking

<sup>27</sup>Es ist . . . ein Misverständniß, im ersten Erscheinen lebender Wesen auf Erden oder auf einem anderen Weltkörper etwas Supernaturalistisches, etwas anderes zu sehen, als ein überaus schwieriges mechanisches Problem. . . . Könnten wir die Bedingungen herstellen, unter denen einst Lebewesen entstanden, wie wir dies für gewisse, nicht für alle Krystalle können, so würden nach dem Principe des Actualismus [fn] wie damals auch heute Lebewesen entstehen.'

<sup>28</sup>Die unwillkürlichen und *nicht nothwendig mit Empfindung verbundenen* Wirkungen der Centraltheile . . . der Stoffwechsel des Gehirnes und Rückenmarkes u. d. m. wären erschöpfend erkannt. Auch die mit geistigen Vorgängen der Zeit nach stets, *also wohl nothwendig* zusammenfallenden materiellen Vorgänge wären ebenso vollkommen durchschaut.' (Emphases added.)

of a tuning fork, and contact with a red-hot iron – du Bois Reymond avers (1898, pp. 42–3):

No mathematically reasoning understanding could determine *a priori* in the two cases, from astronomical knowledge of the material happenings, which process might be the pleasant one, and which the painful one.<sup>29</sup>

But, alas (1898, pp. 40–1; emphasis added):

In so far as the mental processes themselves are concerned, it is evident that even with astronomical knowledge of the mental organ they would be just as ungraspable as they are now . . . Astronomical knowledge of the brain, which is the highest that we can demand of it, reveals to us nothing but matter in motion therein. *Through no conceivable arrangement or movement of little material parts, however, can a bridge be driven into the realm of consciousness.*<sup>30</sup>

Du Bois-Reymond goes on to stress the contrast between, on the one hand, the complete ignorance of mental facts on the part of the astronomical knower of physical facts, and, on the other hand, the complete demystification (*Enträthselung*) of the ultimate problems of the *physical* world, of which such a knower would be capable.

One relatively recent writer whose classic monograph has not quite faded into oblivion, even though it is a source of certain ideas that many a contemporary writer accords much later provenance, is Herbert Feigl, one of Carnap's oldest philosophical comrades. In the *Aufbau*, Carnap had given du Bois-Reymond his proper due (see Carnap, 1928, §166, pp. 266–7, in his discussion of the formulation of the psychophysical problem). As Feigl correctly observed (1958, p. 105), du Bois-Reymond's unsolvable riddle of the universe concerns

the *irreducible* (synthetic) character of the  $\psi$ - $\Phi$  correlations. Wherever we find co-existential or correlational regularities in nature, we hope to find a unitary explanation for them, and in many cases scientific theories have provided fruitful and well-confirmed explanations of this sort. But in the case of the  $\psi$ - $\Phi$  correlations we seem to be confronted with a fundamentally different situation. There is no plausible *scientific* theory anywhere in sight which would explain just why phenomenal states are associated with brain states.

<sup>29</sup>Kein mathematisch überlegender Verstand könnte aus astronomischer Kenntniss des materiellen Geschehens in beiden Fällen a priori bestimmen, welcher der angenehme und welcher der schmerzhaftige Vorgang sei.'

<sup>30</sup>Was nun aber die geistigen Vorgänge selber betrifft, so zeigt sich, dass sie bei astronomischer Kenntniss des Seelenorgans uns ganz ebenso unbegreiflich wären, wie jetzt . . . Die astronomische Kenntniss des Gehirnes, die höchste, die wir davon erlangen können, enthüllt uns darin nichts als bewegte Materie. *Durch keine zu ersinnende Anordnung oder Bewegung materieller Theilchen aber lässt sich eine Brücke in's Reich des Bewusstseins schlagen.*' (Emphasis added.)

That was written in 1958. Let us return, however, to 1880. Du Bois-Reymond went on to concede (1898, p. 69) that:

Excellent thinkers have for a long time recognized that even the simplest sensory experience cannot be rendered comprehensible through any arrangement and movement of matter. I knew very well that false notions on this score were to be encountered widely; but I was almost ashamed to offer such stale beer to German scientists, and hoped only that the novelty of my method of proof might stimulate interest.<sup>31</sup>

Du Bois-Reymond offers his reader a feast of deliciously derogatory counter-critiques directed at those who had reacted to his ideas with dismissive censure. He observes, drolly, (1898, p. 70) that ‘the word *Ignorabimus*’, in which my investigation culminated, really became a kind of *naturphilosophische* shibboleth’.<sup>32</sup>

Indeed, according to du Bois-Reymond’s undoubtedly correct impression of past philosophy, he was by no means alone (1898, p. 70): ‘What I put forward contained nothing which could not have been known by anyone with a degree of familiarity with older philosophical writings.’<sup>33</sup>

The physiologist du Bois-Reymond, whose own speciality was muscles and nerves, would undoubtedly have been familiar with Huxley’s classic work. He cites frequently enough from other scientific writings of Huxley. (In Britain, Huxley’s *Lessons in Elementary Physiology* was so well-known and widely used as a textbook that Lord Shaftesbury, when campaigning for his anti-vivisection bill, criticized it as likely to lead youngsters to undertake their own vivisection experiments on animals.)

### The knowledge argument

Du Bois-Reymond also anticipated the point made more colourfully of late by Frank Jackson’s (1982) tale about the brain scientist Mary.<sup>34</sup> Mary,

<sup>31</sup>‘Daß durch keine Anordnung und Bewegung von Materie auch nur einfachste Sinnesempfindung verständlich werde, haben längst vortreffliche Denker erkannt. Wohl wußte ich, daß über letzteren Punkt falsche Begriffe weit verbreitet seien; fast aber schämte ich mich, den deutschen Naturforschern so abgestandenen Trunk zu schenken, und nur durch die Neuheit meiner Beweisführung hoffte ich Teilnahme zu erwecken.’

<sup>32</sup>‘... das Wort *Ignorabimus*’, in welchem meine Untersuchung gipfelt, ward förmlich zu einer Art von naturphilosophischem Schibboleth.’

<sup>33</sup>‘... meine Aufstellungen enthielten nichts, was bei einiger Belesenheit in älteren philosophischen Schriften nicht jedem bekannt sein konnte, der sich darum kümmerte.’

<sup>34</sup>Even more colourful are the fictional writing exercises hilariously embroidering the character and circumstances of Mary, submitted by the creative writing students to the heroine (or perhaps one should say: female narrative perspective) in David Lodge’s novel *Thinks*.

as the reader no doubt realizes, is possessed of what du Bois-Reymond would call ‘astronomical’ knowledge of the human brain. Du Bois-Reymond wrote (1898, p. 75): ‘Even if we possessed astronomical knowledge of what was going on in the brain, we would not have advanced one hair’s breadth in relation to the coming into being of consciousness.’<sup>35</sup>

Such ‘astronomical’ knowledge of neurophysiology and the physics of colour perception is, of course, precisely what Jackson asks his reader to regard Mary as enjoying before she enjoys her first (and for her, in advance, *unimaginably*) red quale.

So much for supervenience, the mysterian view, the explanatory gap and the knowledge argument. The Germans had anticipated these, more than a century ago. du Bois-Reymond (1898, pp. 48–9) even engages in a *Gedankenexperiment* involving molecule-by-molecule duplicates of Caesar, thereby anticipating Davidson’s Swampman. He even calls them *Doppelgänger*, and writes ‘On our view, Caesar would then be reconstituted mentally as well as bodily’.<sup>36</sup>

## DAVID HILBERT

Let us turn now from the debate in the philosophy of mind over where precisely to draw the line between the knowable and the absolutely mysterious, and consider a related debate over the same difficulty in the philosophy and foundations of mathematics.

Du Bois-Reymond’s *Ignorabimus* was so well-known and so influential in European intellectual circles that it was the foil for David Hilbert’s famous remark in his lecture to the International Congress of Mathematicians in Paris in 1900, in which he posed his famous list of twenty-three mathematical problems:

This conviction that each and every mathematical problem is soluble is a powerful incentive to us in our work. We hear within us the constant call: *There is the problem. Seek its solution. You can find it by pure reason, for in mathematics there is no Ignorabimus.* [Emphasis in original.]<sup>37</sup>

<sup>35</sup>‘Besäßen wir . . . astronomische Kenntnis dessen, was innerhalb des Gehirnes vorgeht, so wären wir in Bezug auf das Zustandekommen des Bewußtseins nicht um ein Haar breit gefördert.’

<sup>36</sup>‘Nach unserer Anschauung wäre dann CAESAR geistig wie körperlich wieder hergestellt.’

<sup>37</sup>‘Diese Überzeugung von der Lösbarkeit eines jeden mathematischen Problems ist uns ein kräftiger Ansporn während der Arbeit; wir hören in uns den steten Zuruf: *Da ist das Problem, suche die Lösung. Du kannst sie durch reines Denken finden; denn in der Mathematik gibt es kein Ignorabimus!*’

See Hilbert (1935, p. 298). Emphasis in original.

Hilbert's optimism in mathematics was a philosophical conviction of long standing. Constance Reid, in her biography of Hilbert (1970, p. 13) writes as follows:

In addition to [Hilbert's and Minkowski's] enthusiastic love for mathematics, they shared a deep, fundamental optimism. As far as science in general was concerned, the period of their university days was one of triumphant pessimism, a reaction against the almost religious belief in the power of science which had flourished in the previous century. The works of Emil du Bois-Reymond, a physiologist turned philosopher, were widely read and much quoted. Du Bois-Reymond concerned himself with the limits of the knowledge of nature – this was, in fact, the title of his most famous lecture. He maintained that certain problems, which he called transcendental, or supersensible, were unsolvable even in principle. These included the nature of matter and force, the origin of motion, the origin of sensation and consciousness. His gloomy concession, 'Ignoramus et ignorabimus' – we are ignorant and we shall remain ignorant – was the catchword of many of the scientific-philosophical discussions of the university. But to both Hilbert and Minkowski such a concession was thoroughly abhorrent.

Hilbert's epistemological optimism, at least in his famous *Problems* lecture, concerns *mathematics* only. And even when he extends his optimism to the natural sciences (see below), it is not in a way that would necessarily contradict du Bois-Reymond's pessimism about the *Welträtsel*. This observation is consistent with David McCarty's independent study (2005) which examines also the relationship between Hilbert and Paul du Bois-Reymond, the mathematician-physicist brother of Emil. The observation was made also by Hermann Weyl, in the seventieth birthday greeting that he wrote for Hilbert in *Naturwissenschaften*. (See also Reid, 1970, pp. 200–1.) Weyl wrote (1932, p. 58):

Nevertheless we do Hilbert an injustice if we lump together his rationalism with that of a Haeckel. In Hilbert's case his view is based on a much more refined concept of knowledge. His attitude would be one of presumptuousness if the knowledge sought were that kind of *magical knowledge* which Faust longed for ("Schau' alle Wirkenskraft und Samen"), which is supposed, by some kind of intellectual intuition, to unlock for us the "inner nature of things", and which even today most people still falsely attribute to science as its goal. This magical knowledge may well have 'existential' meaning for human existence and excite one's feelings, but *that knowledge of reality that proves its worth in predictions* is promoted only by the mathematical method, which "rummages about" in symbols rather than in words, the theoretical construction that is exercised most decisively in physics. [Emphases in original.]<sup>38</sup>

<sup>38</sup>Dennoch tut man HILBERT Unrecht, wenn man seinen Rationalismus etwa mit dem eines HAECKEL in den gleichen Topf wirft. Bei HILBERT liegt ein viel feinerer Begriff des Erkennens zugrunde. Vermessenheit wäre seine Haltung wenn das gesuchte Wissen jene *magische*

We must bear in mind the special sense in which Hilbert regards a problem as soluble. Earlier (see Hilbert, 1935, p. 297) he had explained the mathematician's conviction as to the effect:

that each and every definite mathematical problem must be able to be settled strongly, be this because we succeed in giving an answer to the question posed, or because we show that its solution is impossible and that all attempts to solve it must therefore fail. Suppose one puts forward a definite unsolved problem, such as the question of the irrationality of the Euler-Mascheroni constant  $C$  or the question whether there are infinitely many prime numbers of the form  $2^n + 1$ . No matter how inaccessible these problems appear to us and no matter how cluelessly we confront them at this time – we nevertheless have the certain conviction that they will admit of solution in a finite number of purely logical inferences.<sup>39</sup>

It is clear that for Hilbert a problem can be 'strongly settled' by means of a suitable *impossibility proof*. That is to say, it counts as a solution to show that the problem cannot admit of any 'positive solution'. In this category we could include the later independence results of set theory, such as the proof that the Continuum Hypothesis cannot be proved in ZFC (if ZFC is consistent). Mathematics is distinguished, of course, by its *a priori* character. But it would appear to be compatible with Hilbert's conception of the solubility of problems that one might claim to have settled a problem negatively, by means of intuitive or otherwise *a priori* considerations, in areas other than mathematics but accessible to philosophical methods.

Thus it would be open to a follower of du Bois-Reymond to claim that the riddle of consciousness in the material world has indeed been settled negatively: to wit, that the problem in question will never admit of positive solution strictly within the sphere of the natural sciences. For, the supporter

*Erkenntnis* wäre, nach der sich Faust sehnt ("Schau' alle Wirkenskraft und Samen"), die in einer Art intellektueller Anschauung uns das "Innere der Dinge" aufschließen will und die auch heute noch von den Meisten der Wissenschaft als Ziel untergeschoben wird. Sie mag ihre "existentielle" Bedeutung haben für das menschliche Dasein und das Gefühl beschwingen, aber *das in Voraussagen sich bewährende Wissen um die Wirklichkeit* wird nur gefördert durch die *mathematische Methode*, die zwar nicht "in Worten", wohl aber in Symbolen "kramt", die theoretische Konstruktion, wie sie am entschiedensten von der Physik geübt wird.' [Emphases in original.]

<sup>39</sup>... daß ein jedes bestimmte mathematische Problem einer strengen Erledigung notwendig fähig sein müsse, sei es, daß es gelingt, die Beantwortung der gestellten Frage zu geben, sei es, daß die Unmöglichkeit seiner Lösung und damit die Notwendigkeit des Mißlingens aller Versuche dargetan wird. Man lege sich irgend ein bestimmtes ungelöstes Problem vor, etwa die Frage der Irrationalität der Euler-Mascheronischen Konstante  $C$  oder die Frage, ob es unendlich viele Primzahlen von der Form  $2^n + 1$  gibt. So unzugänglich diese Probleme uns erscheinen und so ratlos wir zur Zeit ihnen gegenüberstehen – wir haben dennoch die sichere Überzeugung, daß ihre Lösung durch eine endliche Anzahl rein logischer Schlüsse gelingen muß.'

will point out, it is not as though continued failure so far is our only evidence for the pessimistic claim. Rather, du Bois-Reymond argued for the impossibility on conceptual grounds, by employing the posit of a Laplacean demon. The demon's cognitive powers are assumed to be perfect; it is a kind of supreme epistemic agency that could be approximated by us only asymptotically, from below. The demon would have unlimited access to singular empirical facts, as well as unlimited deductive power in order to derive predictions from initial conditions and whatever *Weltformel* (world-formula) codified the ultimate laws of nature. Yet even the demon, for all that, would come up against ultimate limits to what he could know. These, du Bois-Reymond argues, are a fortiori the absolute limits to what we mere mortals could know.

Even if it were maintained that this line of argument is too glib an appropriation of the letter of Hilbert's characterization above of what, in general, a solution (positive or negative) to a definite problem might consist in, there would have been no easy transition, even for Hilbert – as Weyl remarked – from such epistemological optimism in mathematics to a corresponding epistemological optimism about the *Welträtsel* of consciousness in the material world. It is worth noting, however, that in his essay 'Naturerkennen und Logik' (see Hilbert, 1935, p. 387), Hilbert extended his optimism so as to include the natural sciences. After quoting Jacobi's claim:

that the sole purpose of all science is the honor of the human spirit and that from this point of view a problem in pure number theory has just as much value as one admitting of applications<sup>40</sup>

Hilbert goes on to say (emphases added):

He who is sensitive to the truth of the noble method of thought and *Weltanschauung* that shines forth from these words of Jacobi will not fall for reactionary and fruitless skepticism; *he will* not believe those who today with philosophical mien and considered tones prophesy cultural ruin, and put up with the Ignorabimus. For *the mathematician* there is no Ignorabimus, and, in my opinion, for the natural sciences also. Instead of the silly Ignorabimus, our solution is quite the opposite:

We must know,  
We will know.<sup>41</sup>

<sup>40</sup>... daß die Ehre des menschlichen Geistes der einzige Zweck aller Wissenschaft ist und daß unter diesem Gesichtspunkt ein Problem der reinen Zahlentheorie ebensoviel wert ist als eines, das den Anwendungen dient.'

<sup>41</sup>'Wer die Wahrheit der großzügigen Denkweise und Weltanschauung, die aus diesen Worten JACOBI hervorleuchtet, empfindet, der verfällt nicht rückschrittlicher und unfruchtbarer Zweifelsucht; *der wird* nicht denen glauben, die heute mit philosophischer Miene und überlegenem Tone, den Kulturuntergang prophezeien, und sich in dem Ignorabimus gefallen. Für *den Mathematiker* gibt es kein Ignorabimus, und meiner Meinung nach, auch für die

A condensed and slightly re-worded extract of this essay was used for Hilbert's radio address from Königsberg on 8 September 1930, on the occasion of his becoming an honorary citizen. (The broadcast is preserved in a 45 rpm recording (Hilbert, 1930).) Because of its brevity, Hilbert's epistemological optimism was expressed even more emphatically in the radio address. It contains the lines '*We must not believe those . . .* [Emphasis added]<sup>42</sup> and '*For us there is no Ignorabimus . . .*'.<sup>43</sup> This insistent optimism, however, still would not touch the *Welträtsel* in question. Du Bois-Reymond might heartily agree with Hilbert that our 'astronomical' knowledge (in the natural sciences – see below) should be pursued, optimistically, as though it were logically complete. This would still (for du Bois-Reymond) fail to speak to the problem of how one is to explain consciousness as a phenomenon in the material world. That is a problem lying not *within* natural science, but *beyond* it – as was clear from the very title of his classic address, 'Über die Grenzen des Naturerkennens'. *By all means*, one can hear the ghost of du Bois-Reymond saying to the ghost of Hilbert, *let us assume that we shall be the equals of the Laplacean demon – WE SHALL KNOW. Still* – and here the sepulchral tone is firm – *the riddle of consciousness remains a riddle. For not even the Laplacean demon can solve it*. Or, to put it in later terminology: even the Laplacean demon is cognitively closed with respect to whatever natural property of the brain it might be (if any) that would explain consciousness.

### EARLY HINTS OF FUNCTIONALISM

Some scholars have recently argued that a doctrine recognizably like modern functionalism (about mind) is to be found in Aristotle's writings. (See, for example, Wilkes, 1978 and Nussbaum & Putnam, 1992.) However, Burnyeat (1992) argues for an opposing view, stressing that Aristotle's conception of the physical was not modern enough to occasion the mind–body problem in the very form to which modern functionalism becomes an intelligible solution (the solution in question maintaining that the mental facts supervene on the physical facts, and that mental functions could be

Naturwissenschaft überhaupt nicht. . . . Statt des törichten Ignorabimus, heiße im Gegenteil unsere Lösung [*sic*]: Wir müssen wissen, Wir werden wissen.'

<sup>42</sup> '*Wir dürfen nicht denen glauben . . .*' (Emphasis added)

<sup>43</sup> '*Für uns gibt es kein Ignorabimus . . .*'. According to Constance Reid (1970, p. 196) in the original wax recording of his radio address, Hilbert can be heard laughing softly after his closing words 'Wir müssen wissen. Wir werden wissen.' The interpretation of that laugh would be all the more enigmatic, given that only the day before Hilbert's radio broadcast (7 September 1930), in Königsberg, during a roundtable discussion at a meeting organized by the *Gesellschaft für empirische Philosophie*, Gödel had made his first public announcement of his first incompleteness theorem for arithmetic. For more details on the immediate reception of Gödel's incompleteness theorems, see Dawson (1985) and Mancosu (1999).

multiply realizable by different physical bases). The aim in this section is less ambitious than the search for a resolution, one way or the other, of this controversy over whether Aristotle may be read as a functionalist in our modern sense. The aim is just to reveal some strong late-nineteenth-century hints of what was to emerge, more fully developed, in the second half of the twentieth century. (Note that the nineteenth century thinkers in question were certainly heirs to that post-Cartesian conception of the physical that Burnyeat was at pains to argue was *not* Aristotle's conception.) We shall find some indication of key developments within the evolving conceptual context – developments that were arguably crucial for the eventual fruition of the twentieth-century statement of functionalism about mind.

Richard Avenarius's short 1876 monograph was mentioned earlier. What is notable about that work, for present purposes, is that on the very first page he speaks of '*die Funktionen der Seele*' (the functions of the mind), of '*die Zweckmäßigkeit der geistigen Organisation*' (the purposiveness of mental organization), and of the '*zweckmäßig fungierende Organisation*' (purposely functioning organization) of a system which is to cope with the problems encountered by a '*leibliche[r] Organismus*' (bodily organism). The mind's '*theoretische Funktion*' is '*das Denken*'. So we see that talk of mental *function* was already in the air.

The next figure to whom attention should be directed is the great developmental biologist and *Naturphilosoph* Ernst Haeckel. Haeckel went interestingly further than the supervenience claim advanced by the sources quoted above. Supervenience is a very broad answer to the question of the mind-body relation. It is presupposed by token-identity theorists, type-identity theorists, epiphenomenalists and functionalists. The differences among these positions concern only the abstract pattern in which mental events, states and processes are 'fixed' by physical events, states and processes; and whether mental events themselves have causal powers. Now, of course, functionalism as a developed philosophy of mind owes much to the analogy with a digital computer. This is needed for the hardware/software contrast, the multiple realizability claim, and the notion of a logical state of a program – for, according to the functionalist, it is with logical states that mental states are to be identified. Thus a single occurrence of the word 'function' will not be much on which to base a priority case; but if it occurs in the right context, it can be seen, in retrospect, as highly suggestive.

This is the case with the extended passage to be quoted below from Haeckel (1899, pp. 13–14). It would appear that Haeckel's choice of the word '*Funktion*' was no accident, and was pregnant with all the right philosophical connotations. The most important one, for present purposes, is that a function in the organismic context is to be understood with a degree of abstractness that enables us to prescind from details of actual physical implementation, when inquiring after its essential nature. This increasing abstractness is compatible with the central sense of 'function' as given in the

Shorter OED: 'The activity proper or natural to a person or thing; the purpose or intended role of a person or thing...'. The main point is that the functions in question should be understood as multiply realizable. The function of the heart is to pump blood; but that very function could have been served by a physiologically different (and differently located) organ, and could even be served by a prosthetic implant.

In further support for this increasingly abstract understanding of 'function', note that it would not be a mere coincidence that, by the time of Haeckel's writing, the *mathematical* concept of a function had been refined to its current state of complete abstraction, as an arbitrary set of ordered pairs that agree on their second members if they agree on their first members. (See the interesting history, culminating in this notion of Dirichlet, in Maddy, 1997.) To the extent that *mathematical* functions could be 'alinguistic', that is, not tied down to expression by any linguistic rule, so too could higher *organic* or *cerebral* functions be 'abiological', that is, not tied down to expression (or realization) by any particular anatomical structures. That is to say, it would not be anachronistic for one to claim that such cerebral functions could well have been understood, even in Haeckel's day, as multiply realizable.

There is another source of increasingly abstract understanding of these matters in the period in question: the profound advances made by Hilbert in our understanding of axiomatic theories.<sup>44</sup> Hereafter, the two threads in the foregoing discussion (concerning what is knowable about the mind, and what is knowable about the objects of mathematics) become *conceptually* intertwined, after having shared an historical source (even if not featuring together) in the epistemological pessimism of du Bois-Reymond. It was Hilbert who first developed our understanding of a set of axioms as implicitly defining the theoretical primitives involved; and who stressed that as far as matters of interpretation were concerned, all that counted was *structure*, rather than the actual individual objects involved. A geometrical theory, for example, could just as well be interpreted as being about tables, chairs and beer mugs, as about points, lines and planes. Thus the theoretical 'objects' involved could be abstracted out as mere 'loci of relational intersections'. This mind-set no doubt contributed decisively to philosophers' later construal of mental states as logical states of a programme, which could be identified by their 'place within a transitional network', rather than by the physical condition of whatever biological organ or substrate produced the mental functioning in question. Function itself thus was able to become a matter of form, rather than of contained matter or material content.

There is of course much more to modern functionalism than merely the claim that mental states are functions of (parts of) the brain. Essential to the contemporary view is the conception of the functional programme as

<sup>44</sup>Here the author is indebted to Wilfried Sieg.

mediating between sensory input and behavioural output, with the state-transitions affected also by the influence of other internal states themselves. Still, these further sophistications presuppose an original ‘breakthrough insight’ – in much the same way that today’s eliminative materialist or biological essentialist needed to build on the breakthrough insight of a basic materialism about mind in the first place.

It is in the interests only of relative completeness that the promised passage from Haeckel be given, which perhaps ought to earn him the title ‘grandfather of functionalism’ in the philosophy of mind. It is offered with all the qualifications just made explicit; and it is to be hoped that the reader will not regard it as evidence of a misguided over-enthusiasm in detecting all manner of precedents in the literature of *Naturphilosophie*. Not only can the following passage be read as an early statement of the essential *basic* claim of functionalism about the mental; it is also an explicit anticipation of the basic principles of Quinean naturalized epistemology.

It is interesting to note that this book sold almost a quarter of a million copies around the beginning of the twentieth century, and was translated into fifteen languages.<sup>45</sup>

SOLUTION OF THE RIDDLES OF THE UNIVERSE. The means and ways which we have to pursue in order to solve the great riddles of the universe, are none other than those of pure scientific knowledge itself, that is to say, first, experience, and second, inference. We acquire scientific experience through observation and experiment, whereby it is predominantly our sensory organs, and secondarily the ‘inner sense-centers’ of our cerebral cortex that are active. The microscopic elementary organs of the first are the sensory cells, while those of the second are groups of ganglion cells. The experiences which we have attained of the external world through these inestimable organs of our mental life are then converted into ideas by other parts of the brain, and these ideas in turn are concatenated by association to form inferences. The formation of these inferential transitions takes place in two different ways, which I am convinced are equally valuable and indispensable: *induction* and *deduction*. Further developed brain operations, the formation of chains of reasoning that hang together, abstraction and concept-formation, the expansion of cognitive understanding by means of the plastic activity of imagination, and finally consciousness, thinking and philosophizing, *are just as much functions of the ganglion cells or neurons of the cerebral cortex* as are the foregoing simpler mental activities. We unite them all in the overarching concept of Reason. (Last emphasis added.)<sup>46</sup>

<sup>45</sup>See the *Nachwort* in the 1908 edition.

<sup>46</sup>Lösung der Welträtsel. Die Mittel und Wege, welche wir zur Lösung der großen Welträtsel einzuschlagen haben, sind keine anderen als diejenigen der reinen wissenschaftlichen Erkenntnis überhaupt, also erstens *Erfahrung* und zweitens *Schlußfolgerung*. Die wissenschaftliche Erfahrung erwerben wir uns durch Beobachtung und Experiment, wobei in erster Linie unsere Sinnesorgane, in zweiter die “inneren Sinnesherde” unserer Großhirnrinde tätig sind. Die mikroskopischen Elementarorgane der ersteren sind die Sinneszellen, die der letzteren Gruppen

## SUMMARY

The primary purpose of this historical survey has been a limited one: to establish the claim that eminent figures in German *Naturphilosophie*, along with their Victorian counterparts across the Channel, had already given reasonably complete expression to several of the major themes of contemporary analytical and scientifically informed philosophy of mind. Various nineteenth-century figures championed quite explicitly hardline versions of materialism, supervenience and rudimentary functionalism, as well as mysterianism and the explanatory gap. Moreover, these figures were not obscure or understandably neglected. They included the most eminent physiologists and philosophers of their times, in positions of public influence, and with a penchant for popularization. In other words, their influence was widely felt, and ought to have been longer lasting. Their neglected works appear to provide a rich source of material ripe for reconsideration by anyone hoping to contribute to discussion of the mind–body problem in the context of modern science.

A secondary concern has been to describe the influence of the intellectual climate after the *Ignorabimusstreit* on the philosophy and foundations of mathematics, exemplified in the pivotal figure of David Hilbert in the early twentieth century. Increasingly abstract conceptions of mathematical functions and structures mirrored those in biology and psychology. The more abstract conception of function allowed twentieth-century philosophers to revisit the problem of rational thought and language (the sixth problem on du Bois-Reymond's list of seven) and, in the spirit of Haeckel, offer a broadly functionalist account of cognition and linguistic understanding that is fully comprehensible within a monistic physicalistic framework. The fifth problem, the problem of (sentience and) consciousness, however, remains as intractable as du Bois-Reymond insisted it would.

It is frequently the case that our grasp of an actual problem or issue, and of the value of a particular insight or idea, is enhanced by an appreciation of the immediate *Zeitgeist* – scientific, philosophical and theological – in which the insight or idea receives its first identifiable expression. It is not being

von Ganglienzellen. Die Erfahrungen, welche wir von der Außenwelt durch diese unschätzbarsten Organe unseres Geisteslebens erhalten haben, werden dann durch andere Gehirnteile in Vorstellungen umgesetzt und diese wiederum durch Assoziation [*sic*] zu Schlüssen verknüpft. Die Bildung dieser Schlußfolgerungen erfolgt auf zwei verschiedenen Wegen, die nach meiner Überzeugung gleich wertvoll und unentbehrlich sind: *Induktion* und *Deduktion*. Die weiteren verwickelten Gehirnoperationen, die Bildung von zusammenhängenden Kettenschlüssen, die Abstraktion und Begriffsbildung, die Ergänzung des erkennenden Verstandes durch die plastische Tätigkeit der Phantasie, schließlich das Bewußtsein, das Denken und Philosophieren, sind ebenso Funktionen der Ganglienzellen oder Neuronen der Grosshirnrinde wie die vorübergehenden einfacheren Seelentätigkeiten. Alle zusammen vereinigen wir in dem höchsten Begriffe der Vernunft.' (Last emphasis added.)

(Haeckel, 1899, pp. 13–14)

denied that modern writers have provided new insights, or applied recently invented methods or concepts in illuminating ways. (It would be an intolerable state of affairs were this not the case.) The suggestion is just that one needs to be mindful of the essential kernels, and original sources, of what might otherwise, mistakenly, be taken to be wholly contemporary doctrines. As Collingwood once wrote, concepts, like individuals, have histories. We know and understand them better for having a more intimate acquaintance with the backgrounds that brought them forth. This is not the occasion to argue for this hermeneutic contention. It is not unreasonable, however, to rely on it as a methodological presupposition in undertaking the explorations above. As Richard Sorabji has written (2005, p. 34),

The possibility of reviving ideas is part of what gives point to philosophers studying the history of philosophy. It liberates us from the circle of ideas which happen to be most recent and expands the philosophical imagination.<sup>47</sup>

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<sup>47</sup>One of Sorabji's concerns in the piece cited here is to show that contemporary theories of personal identity provide further instances of the recurrence (dare one say 'eternal' recurrence?) of ideas.

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