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IS THIS A PROOF I SEE BEFORE ME?

By NEIL TENNANT

COLIN MCGINN's paper 'Truth and Use' (M. Platts, ed., *Reference, Truth and Reality*, Routledge and Kegan Paul, 1980) presents a case against anti-realism of the kind espoused by Dummett. I believe the argument to be flawed in two respects. First he overstates the anti-realist view on what it is to grasp the meaning of a sentence. Secondly he is far too generous a radical interpreter in drawing realist conclusions from the imagined behaviour of some strange creatures he describes. I shall deal with these points in turn, but shall have to refrain here from exploring a theme suggested by some of McGinn's remarks—that of developing a suitably anti-reductionist account of meaning that could support realism. My purpose here is to be tersely critical rather than constructively expansive of McGinn's views.

The following is McGinn's statement of what he takes to be Dummett's view of what grasp of meaning consists in:

Understanding a sentence . . . consists in an ability to determine its truth value in some canonical way, and acquiring a mastery of a language consists in acquiring such capacities with respect to its sentences. In fact, such a capacity is both executive and recognitional: it consists in a disposition, when prompted, to undertake a procedure which, in a finite time, terminates in a recognition, signalled by some overt gesture, that the sentence's truth condition is fulfilled or not fulfilled, as the case may be. It is because (actual) possession of such an effective decision procedure for truth value is a practical capacity that it provides the needed link between knowledge of sense and use (loc. cit. p. 21).

The misconception built into this straw account is the following. No intuitionist or anti-realist would demand that grasp of meaning should consist in an ability to decide truth value. What they maintain rather is that grasp of meaning consists in an ability to decide, *of any particular presentation*, whether it establishes the sentence as true or false. Here by "presentation" I mean something like a purported proof in mathematics; or an orderly series of pointings and indicatings in simpler ostensive situations; or a series of experiments, recorded observations and measurements in the natural sciences, documented and presented to the would-be decider so that he can check whether it establishes the result claimed.

McGinn's account has the following implication, which reduces it to absurdity. On his account, no intuitionist grasps the meaning of Goldbach's conjecture, which can be expressed in a not too complicated sentence of first order arithmetic. Now we do not know whether Goldbach's conjecture is true or false or undecidable on the basis of

presently accepted axioms. We do not know whether we might not come across a proof of it in the future, or discover an isolated counter-example—that is, an even number greater than two that is not the sum of any two primes. We do not know whether a proof, if and when discovered, will require inexplicable ingenuity or will be seen, with the benefit of hindsight, as something that we must have been really dull not to have come up with before. All these things we do not know, and more. But this abyss of ignorance about Goldbach's conjecture does not—indeed cannot—divest it of determinate meaning for the intuitionist. Just because the intuitionist rejects the classical notion of truth—the notion of a sentence being determinately true or false, independently of our ability to recognize it as such—does not mean that he throws the Fregean baby out with that murky bathwater. The Fregean baby of course is the insight that we grasp meanings of sentences by grasping the meanings of their parts and also the way they are put together. *Of course* an intuitionist grasps the meaning of Goldbach's conjecture! For he grasps the meanings of its primitive parts and of the logical operators involved in its construction. His grasp of the former is displayed in his mastery of the recursive rules for plus and times. His grasp of the latter is displayed in his knowledge of what in general constitutes a proof of a sentence with the appropriate logical operator dominant.

We must distinguish between a sentence's having a determinate truth *value*, and having a determinate meaning, or truth *conditions*. McGinn is asking too much of the intuitionist in requiring that this very competent grasp, as just described, of the meaning of a sentence such as Goldbach's conjecture should underlie, or entail, or give rise to or inevitably be accompanied by an ability to evaluate it—that is, to get to work on it with an effective method that will terminate in a decision as to its truth value. This vitiates the following point in his case against Dummett:

. . . a conception, encouraging assertion of bivalence, of a sector of reality that transcends . . . recognitional powers . . . could not be that in which . . . grasp of the sense of the relevant sentences consists; for, by Dummett's argument, nothing can qualify as *knowledge of meaning* unless it relates directly to some linguistic disposition of the speaker—a disposition, we have been agreeing, *to engage in a truth-value deciding procedure*—and the conception in question precisely does not relate to any such disposition (loc. cit., p. 29; my emphases).

McGinn is clearly demanding too much. The intuitionist may hit upon a proof after inspiration and creative work, the general lines of which may with hindsight be explained by appeal to his grasp of various aspects of the conclusion's meaning. Grasp of meaning no doubt somehow guides one's search for proofs, but it is only the proof *once found* that licenses an effectively defensible claim to *knowledge of truth value* (contrast with the quote above)—as I have argued in 'Language Games and

Intuitionism', *Synthese* 42, 1979, pp. 297-314. Grasp of the meaning does not guarantee a decision as to truth value, even on the intuitionistic account. As an intuitionist I could argue thus: 'I know what Goldbach's conjecture means because I know what it would be to have a proof of it before me. Don't ask me to tell you what that proof looks like! (That is, don't ask me for a, or the proof—for there may not be one.) To test whether I know, as I claim I do, what it would be to have a proof of Goldbach's conjecture before me, you could find one yourself and then have me check it; but that of course is not the only way.'

Indeed, we are not forced to ascertain grasp of meaning in this way only. Why can we not simply rely on the intuitionist's general understanding of his language, the language to which Goldbach's conjecture belongs? We have no right to insist that grasp of meaning be confirmed sentence by sentence, that we present for inspection proofs or disproofs of each and every sentence for which we raise the question whether meaning has been grasped. The intuitionist has compositional capacities like those of the classicist. He can understand individual words and operators and grasp sentential pedigree. And this allows him to grasp the meanings of new sentences, including Goldbach's conjecture. Moreover, the "basic grasps" involved can be ascertained by investigating his general ability to infer conclusions, reduce proofs to canonical proofs, find proofs of simple theorems, etc. etc.

In conclusion, then, I repeat: it is too much to require of the anti-realist that he display grasp of sentential meaning by successfully deciding the sentence. The anti-realist can admit the possibility of definitely meaningful yet undecidable sentences. McGinn does not attend closely enough to this distinction between being able to decide and being able to check that which purportedly decides.

I wish now to raise a further objection, this time against McGinn's "model case" in support of realism. He describes a thought experiment designed to show that realism is not only correct, but virtually unavoidable. Briefly, his imagined "community" of creatures is one of individuals rooted to the spot on the north side of a mountain. They cannot, "*in principle*", investigate the south side. Yet we are asked to suppose that they are possessed of a first order language with the operator S, taken to mean 'On the south side of the mountain it is the case that . . .' We are asked further to suppose that, 'like us, they are disposed, realistically, to assert bivalence across the board'. Nevertheless, continues McGinn, 'we have no account (from the anti-realist) of what it is about their use of language that confers upon their sentences the kind of sense a truth-conditions theory purports to record'. He then asks us to concede that 'the mountain-dwellers can manifest their knowledge of transcendent truth-conditions . . . by interpreting the assertions of fellow speakers as expressions of . . . realist beliefs'.

At first glance this sounds no different from the usual realist gestures at the widespread assertion of bivalence as part of the use of language that must be legitimated by any "meaning is use" theory. But the anti-realist's rejoinder here is well known. He considers rather some central feature of use on which to erect his theory, which can then enjoin peripheral reform. (Prawitz has made this point ably in his paper 'On the controversy between classical and intuitionistic logic', *Theoria*, 1979.) What is so different about the mountain-dwellers' case that they should be beyond this injunction, secure against any allegation of being philosophically muddled?

A great deal would seem to turn here on McGinn's claim (implicit in the case-description, but a claim that is nevertheless more in the realm of diagnosis than of listing symptoms) that the only or the best possible translation into English of the operator S in the language of the mountain-dwellers is the one given. On the one hand he appeals to their utter immobility as a limitation *in principle* on their capacity to apply decision procedures with respect to sentences containing S. But on the other, he appeals to their use of such sentences in order to attribute to them realist beliefs about what is the case on the south side of the mountain.

Now McGinn cannot have it both ways. There is a tension between these two aims, just as there is a tension between the two aims of finding a scientific theory that is as simple as possible, yet at the same time most strongly supported by the data (cf. Elliott Sober, *Simplicity*, Clarendon Press, 1975). I would argue that the case as McGinn describes it does not—at least, not without considerable expansion of pre-emptive detail that he does not provide—exclude translations of the operator S consonant with a non-realist account of the beliefs expressed by the sentences; beliefs that are now different, given the different translation of S, from those McGinn so readily attributes. The line of argument I am suggesting here is reminiscent of Hacking's in 'The Identity of Indiscernibles', *Journal of Philosophy*, 1976. Hacking argues that thought experiments about impoverished universes allegedly containing non-identical indiscernibles can always be countered by re-describing them, consistently with all the legitimate constraints imposed by their inventors, in such a way as to simplify their structure and make them conform to Leibniz's Law. No universe *has* to be described in such a way as to violate that law, and every universe *can* be described in such a way as to preserve it. (For example, Ayer's serial universe of sounds . . . ABCDABCD . . . can be re-described as the universe

A B
D C

with circular, clockwise (I) time.)

Now the same applies to McGinn's 'community' of rooted conjecturists. If we regard them as *in principle* incapable of south-side routines, what supports our attribution to the operator S of the meaning 'On the south side of the mountain it is the case that . . .'? The principle of charity can't license such a hand-out. McGinn's only supporting argument is ' . . . a conception of that (for them) inaccessible reach of reality seems forced upon them if they are to have any reasonable explanation of the things they do observe: sheep disappearing and reappearing, etc.' How so? How can we, confronted by beings so crashingly non-paripatetic, attribute to them even a conceptual scheme of continuants? Given the constraints on their own movement, on re-orientation and measurement, on sizing up and sorting out, on separating, classifying, discriminating, manipulating, how can we even begin to attribute to them that conception of space similar to ours, against the background of which alone the thought fragment 'On the south side of the mountain it is the case that . . .' could be grasped?

If McGinn were to answer by expanding further on the capacities of his mountain-dwellers, one would expect the increased behavioural repertoire to sustain two concessions. First, it would underpin richer translations and richer attributions of belief to the mountain-dwellers, which would help McGinn. But secondly—and this would detract critically from the first concession—it would also undercut the alleged limitation on ability *in principle* to perform (in paradigmatic cases, as emphasized above) the kinds of procedure mastery of which underpins grasp of sense. The anti-realist insists that if we can show that a creature possesses certain beliefs by virtue, among other things, of grasping certain meanings, then, within the same realm of behaviour on the basis of which we show that, the creature must be able effectively to defend his assertions. The credit extended by charity should always earn this sort of interest.

An objector might at this point raise the problem of specifying precisely what it is about the abilities I have described the intuitionist as needing that distinguishes him from the classicist, or realist. For do not classicists also check proofs competently, and thereby display the same sort of ability? Certainly they do; but their proofs, which in general allow the classical moves of excluded middle, or double negation, or classical reductio or dilemma, to not provide them with recipes for the *effective* defence of their truth claims. The intuitionist, however, dealing as he does only with canonical proofs and effective procedures for reducing proofs to canonical form, *can* use his proofs in this way. *This* is the critical location of the emphasis on mastery of effective procedures, that I have been arguing is misplaced by McGinn.