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IS EVERY TRUTH KNOWABLE? REPLY TO HAND AND KVANVIG

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Michael Hand and Jonathan Kvanvig argue that the author's case for the knowability of truth presented in Chapter 8 of *The Taming of The True* is vitiated by the allegedly *ad hoc* nature of a restriction there proposed on the Knowability Principle, to so-called Cartesian propositions. In this note I rebut their arguments.

I. Does the Fitch Proof Pose a Real Problem for the Anti-realist?

In Chapter 8 of *The Taming of The True*[2] (henceforth: *TToTT*) I analyzed the famous argument, due to Fitch, that purports to derive absurdity by combining some general normative principles concerning the epistemic operator 'knows that' with two main claims, to wit

1. All truths are knowable;

2. Not all truths are known (or the intuitionistically stronger, but classically equivalent: Some truths are unknown).[3]

Michael Hand and Jonathan Kvanvig[4] claim that 'Tennant grants the validity of the *reductio* and agrees with realists that (1) should be denied.' This is not so. It is one thing to analyse in detail the precise logical structure of the would-be *reductio*; yet it is another thing entirely to grant its validity (in a way that carries with it recognition that a crisis of thought has been precipitated). To be clear for the record: I do not grant the validity of the would-be *reductio* in the important sense just clarified. Even though I grant the formal correctness of the *reductio*, I am more inclined to refrain from asserting (2) than I am inclined to refrain from asserting (1).

The advantage of setting out in detail the precise formal structure of the would-be *reductio* as I did in the text[5] is that one has a complete inventory of every premiss and step of inference that might be called into question by one who refuses to accept the result. Among those premisses and steps, in the present instance, are the following.

[1] I am grateful to Joseph Salerno and an anonymous referee for comments on an earlier draft.
[2] Oxford: Clarendon Press, 1997.
[3] In *TToTT* the expression $K\varphi$ was defined as the abbreviation of $\exists x \exists t\ xK_t\varphi$—that is, someone at some time knows that φ. Thus when one says that not all truths are known, one does not mean that not all truths are currently known. Rather, one means that not every truth will at some time be known by someone.
[4] 'Tennant on Knowability', *Australasian Journal of Philosophy* 77 (1999), pp. 422–28; at p. 422.
[5] Op. cit., pp. 260–1.

1. the realist premiss that some truths are unknown; (or, what is classically, but not intuitionistically, equivalent:[6]

2. the 'anti-actualist' premiss that not all truths are known);

3. the anti-realist premiss that all truths are knowable;

4. the application of the rule of *classical reductio* whereby one infers $K\varphi$ after reducing the assumption $\neg K\varphi$ to absurdity.

For the fully formal proofs that make the foregoing comments clear, the reader is referred to *TToTT,* ch. 8. To summarise here: first, there is a proof (say Π) of $\bot$ (absurdity) from the premisses $\psi \wedge \neg K\psi$ and $\forall\varphi(\varphi \rightarrow \Diamond K\varphi)$. This proof Π, moreover, does not involve the factive inference '$K\varphi$, therefore φ'. Two different proofs can now be constructed, using π as a sub-proof. If one wishes to construct a Fitchian *reductio* using the (intuitionistically stronger) realist premiss instead of the (classically equivalent) anti-actualist premiss, then one can proceed as follows:

$$\dfrac{\exists\varphi(\varphi \wedge \neg K\varphi) \qquad \begin{array}{c} {}^{(2)}\overline{\underbrace{\psi \wedge \neg K\psi, \forall\varphi(\varphi \rightarrow \Diamond K\varphi)}} \\ \Pi \\ \bot \end{array}}{\bot}\,(2)$$

This, indeed, is the form of Fitch's *reductio* that is most often set out by realists. The gross form of this reductio is proffered, for example, by Hand and Kvanvig[7] when introducing their discussion of my treatment of the Fitch paradox, even though that treatment revealed the paradox as arising from the use of strictly weaker rules and assumptions than they themselves used. Once again, we need to inquire whether the anti-realist is bound to find the *reductio* proof compelling. This is a different matter from inquiring after its *validity*. Even if valid, the *reductio* proof can fail to be compelling (can fail to precipitate a crisis of thought) simply because it may use a premiss that the anti-realist would never be willing to assert. And such is the case with the stronger, realist premiss to the effect that *there is an unknown truth*. As I made clear in my text,[8] 'even the soft anti-realist should be wondering what examples of truths of the form $(\varphi \wedge \neg K\varphi)$ might be given.' The claim under discussion is an existential claim—*there is* an unknown truth. From the anti-realist point of view, one would never be warranted in asserting this claim. For, in order to assert such an existential, one needs to be able to provide a 'witness'. But in providing a witness

[6] That some truths are unknown implies intuitionistically (hence classically) that not all truths are known. But that not all truths are known does not intuitionistically imply that some truths are not known—even though the implication holds classically.

[7] Loc. cit., p. 422. I say 'gross form' because the internal details of their *reductio* proof differ from those of mine. They appeal directly to the elimination inference '$K\varphi$, therefore φ', which is factive. By contrast, I make no use of any factive inference involving *K.* This matter is taken up in more detail in Part II below.

[8] Op. cit., p. 265.

ψ, and in doing so showing that $(\psi \wedge \neg K\psi)$, one will have to show that ψ. At that point one will know that ψ; whence it ought to be impossible to proceed further with the proof of the second conjunct to the effect that ψ is *not* known (by anyone, at any time)!

If one wishes to construct a Fitchian *reductio* by using the weaker anti-actualist premiss rather than the realist premiss, then one can proceed as follows:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\underbrace{\dfrac{\overset{(3)}{\psi} \quad \overset{(2)}{\neg K\psi}}{\psi \wedge \neg K\psi}, \forall\varphi(\varphi \rightarrow \Diamond K\varphi)}_{\Pi}}{\bot}}{K\psi}\,(2)}{\psi \rightarrow K\psi}\,(3)}{\forall\varphi(\varphi \rightarrow K\varphi)} \quad \neg\forall\varphi(\varphi \rightarrow K\varphi)}{\bot}
$$

The anti-realist will allow the step (marked (2) in the last proof) of apparently classical *reductio ad absurdum,* provided that its conclusion $K\psi$ is decidable. As was argued in *TToTT,* at pp. 262–3, this will be the case if and only if ψ itself is decidable.[9] The question for the anti-realist, then, when confronted with the foregoing *reductio* proof, is whether he should endorse the anti-actualist premiss $\neg\forall\varphi(\varphi \rightarrow K\varphi)$, on the tighter reading called for in the proof-theoretic context. That is to say, the question is whether the anti-realist should deny that every *decidable,* true proposition is known (by someone, at some time).

It is not at all clear that such a denial would be the right way for the anti-realist to go. In *TToTT,* I was careful to avoid denying that every decidable, true proposition is known. Nor, however, did I assert that every decidable, true proposition is known.

Still, an interesting case for this claim might well be made out. For a decidable proposition is one for which there is a decision procedure; and the whole point of a decision procedure is that it takes one surely to a correct verdict. So if the proposition is, as we are supposing, true, then that verdict will be to that effect. Thus in some sense we already know its truth, even if we might not be able to render a verdict immediately, on the basis of the occurrent contents of our minds. This sense of 'knows' is what epistemic logicians call the virtual or implicit sense. It is weaker than that of 'occurrently knows'. But the advantage of the virtual sense is that it affords us the rudiments of a logic for the epistemic relation $xK_t\varphi$. The virtual sense allows one to say that if one possesses (at time t) an effective method for determining an answer to a question (i.e., a canonical proof or refutation of the proposition φ involved), then one knows the answer at t. Even while conceding that such an approach to the notion of knowledge (i.e., using K with the virtual sense) might amount to conceptual reform, it is worth pointing out that such a reformed notion of knowledge could still feature in a knowability principle that would provide just

9 We must bear in mind that throughout this discussion, as in Chapter 8 of *TToTT,* the formal sentence $K\varphi$ is short for 'Some thinker, at some time, knows that φ.' It does not mean 'Some thinker knows now that φ.' Our reading makes the Fitch paradox just as much of a problem as it would be on the 'now' reading, and indeed makes it *prima facie* easier for the opposition to prove such a paradox.

as critical a bone of contention between the realist and the anti-realist as does the present form, in which 'x (at t) knows that φ' tends to be read more narrowly, with an occurrent flavour.

The conclusion of the preceding discussion is that I do not grant the efficacy of either form of the Fitchian *reductio* proof. Neither of them precipitates a crisis in thought for the anti-realist.

Strictly speaking, I do not grant that the knowability principle (in the form that all truths are knowable) ought to be denied. Nevertheless, in Chapter 8 of *TToTT* I followed a restriction strategy—a strategy involving a principled re-formulation of the knowability principle so that it would retain its anti-realist bite, but not allow realists to go barking up the wrong tree. In the remainder of this response, I intend to show that Hand and Kvanvig have seized on the wrong bone to pick with the anti-realist.

II. Is the Restriction of the Knowability Principle to Cartesian Propositions *Ad Hoc*?

Here is a general claim worthy of careful consideration:

> No proposition provably inconsistent with the general thesis that all propositions have a certain property infirms our right to say, substantively, informatively, and importantly, that the property is nevertheless possessed by any proposition not provably inconsistent with the general thesis.

Put another way, retaining utmost generality, we have the following pattern of thesis restriction:

> Thesis: $\forall\psi\Phi(\psi)$. Putative counterexample: φ
> Reason for saying it is a counterexample:
> $$\forall\psi\Phi(\psi),\ \varphi \vdash \bot \text{ (and both premisses are needed)}$$
> Restricted Thesis in response to putative counterexample:
> $$\forall\psi(\neg[\forall\theta\Phi(\theta),\ \psi \vdash \bot] \rightarrow \Phi(\psi))$$
> The Restricted Thesis can be substantive, informative and important. The objection that the restriction invoked is *ad hoc* is groundless.

Here is a famous historical example of the successful restriction of an important thesis about propositions:

> Thesis: $\forall\psi(\psi \Leftrightarrow \mathit{True}(\psi))$
> Putative counterexample (Epimenides): This sentence is false
> Reason for saying it is a counterexample:
> $$\forall\psi(\psi \Leftrightarrow \mathit{True}(\psi)),\ \text{This sentence is false} \vdash \bot$$
> Restricted thesis in response to putative counterexample:
> $$\forall\psi(\neg[\forall\theta(\theta \Leftrightarrow \mathit{True}(\theta)),\ \psi \vdash \bot] \rightarrow (\psi \Leftrightarrow \mathit{True}(\psi)))$$
> The Restricted Thesis about truth is substantive, informative and important. The objection that the restriction invoked is *ad hoc* is groundless.

The restricted thesis about truth to which almost every philosopher subscribes is in fact *even more restricted* than the so-called Restricted Thesis just given. Tarksi's resort to language-levels in effect renders the scope of the universal quantification in the thesis less ambitious than that of the Restricted Thesis countenanced here. The latter, for example, could apply by Universal Instantiation to the truth-teller sentence 'This sentence is true', which lies beyond the scope of Tarski's theory of truth. In seeking to avoid the Liar paradox, and stratifying languages so that the truth-predicate for a given language could not be part of the very same language, Tarski can hardly be accused of making an *ad hoc* restriction to his disquotational Thesis about truth.

With that much by way of background and general motivation, we come to the topic of disagreement between Hand and Kvanvig, and the author.

Thesis (Knowability Principle): $\forall\psi(\psi \rightarrow \Diamond K(\psi))$.
Putative counterexample (Fitch): There is an unknown truth
Reason for saying it is a counterexample:

$$\forall\psi(\psi \rightarrow \Diamond K(\psi)), \text{There is an unknown truth} \vdash \bot$$

Restricted Thesis in response to putative counterexample:

$$\forall\psi(\neg[\forall\theta(\theta \rightarrow \Diamond K(\theta)), \psi \vdash \bot] \rightarrow (\psi \rightarrow \Diamond K(\psi)))$$

The Restricted Thesis about knowability is substantive, informative and important. The objection that the restriction invoked is *ad hoc* is groundless.

Indeed, the restriction proposed on the Knowability Principle in Chapter 8 of *The Taming of The True* is even weaker than would still be justifiable on this general approach to restrictions. The actual restricted thesis in contention is that all Cartesian propositions, if true, are knowable:

$$\forall\psi(\neg[K\psi \vdash \bot] \rightarrow (\psi \rightarrow \Diamond K\psi))$$

The deducibility relation that features in the antecedent is that of the appropriate epistemic and modal logic governing the *epistemic and modal* operators that might (in addition to K) occur within ψ. Note that we define 'φ is Cartesian' to mean that $K\varphi$ is not provably inconsistent.

When the anti-realist advances this (restricted) thesis, the modal suffix in the word 'knowable' is to be understood by reference to the capacities of rational beings that finitely extend ourselves. This of course makes it harder for the knowability principle to be true. For now there can be no recourse to an omniscient God, in order to secure knowability trivially. That φ is knowable does not mean that some being or other (including, perhaps, an omniscient God) could know that φ. Rather, it means that some rational being *whose perceptual and intellectual faculties only finitely extend our own* could come to know that φ. This idealization is to allow for the obvious cases where, say, the shortest mathematical proof of a proposition is too long to be surveyed by a human being within a lifetime. It allows us to overcome what Russell called 'merely medical' limitations.[10]

[10] For further discussion of 'knowability in principle', and its explication by reference to knowers whose capacities only finitely extend our own, see *TToTT,* ch. 5.

It is important to appreciate how even the restricted thesis about knowability is a genuine point of contention between the realist and the anti-realist. The realist is willing to assert that even among the Cartesian propositions, there could well be propositions that are true, but whose truth might in principle elude detection, whether by us *or by any finite extensions of ourselves.* This is a modal claim (adverting to possible existence), and of course it conflicts with the correspondingly modalized claim of the anti-realist: that it is necessary that all propositions whose being known is not provably inconsistent will, if true, be knowable. It is difficult to understand how a realist could rest content with 'refuting' the unrestricted knowability principle by appeal to a very dubious counter-example, without being moved to engage the anti-realist further on the more restricted domain of propositions φ such that not-[$K\varphi \vdash \bot$]. The realist who insists that the restricted knowability principle is *ad hoc* has not joined the argument at its crux.

Suppose, for example, that it had been an anti-realist, setting out to write a seminal article on truth as epistemically constrained, who had first discovered the argument that now carries Fitch's name. Suppose further that this anti-realist accordingly proposed only the following thesis about knowability: all propositions involving no semantic predicates or propositional attitudes are, if true, then knowable. This would have been completely analogous to Tarski's discussion of the Liar in his famous paper on truth, and his subsequent attempt to provide a formally correct and materially adequate theory of truth only for languages that were not semantically closed.

Ad hoc emendations to general laws in natural science are usually a sign that Nature has proved to be a little recalcitrant. *Ad hoc* restrictions on the scope of a general physical law detract from its applicability. The physical situations excluded by the restriction will still exist, and the physical magnitudes constituting those situations will still evolve through time; but by dint of the *ad hoc* restriction on the law(s), one's theory will have nothing predictive or explanatory to say about those situations.

In conceptual analysis and philosophical clarification, however, matters are different. What appears to be an *ad hoc* restriction of an important principle can, on closer inspection, turn out to be no such thing. One needs to enquire whether the restriction is framed in such a way as to latch onto something quite general, which one can understand as thwarting the unrestricted principle—but in ways that do not speak to the applicability of the main predicate of the principle, and the philosophical significance that can be attached to such applicability. Such is the case with the proposed restriction of the knowability principle.

Hand and Kvanvig did not see that this restriction was substantive and independently motivated, because they paid no attention to the discussion in section 3 of Chapter 8 of *TToTT* (pp. 252–9), titled 'On Wondering Whether'. The aim of that lengthy discussion was to show how a result directly analogous to the Fitch paradox would arise for the operator 'wonders whether' in place of 'knows'. The first theorem was that $W(\varphi \wedge \neg W\varphi)$ is logically inconsistent. This led to the second theorem, which was that if φ is not logically true, then φ is logically inconsistent with $\neg W\varphi$. The discussion showed that 'there are logical limits on that towards which we can rationally hold our attitudes.' (p. 258) This motivated a non-*ad hoc* restriction of the earlier apparently plausible principle to the effect that one can (rationally) wonder of any contingent proposition whether it is true. The extra condition to be imposed, over and above contingency, is that $W\varphi$ should be consistent. Our earlier pattern of thesis restriction now has another instance:

Thesis (Curiosity Principle): $\forall\psi((\text{not-}[\psi \vdash \bot] \wedge \text{not-}[\vdash\psi]) \rightarrow \Diamond W\psi)$
Putative counterexample:

(γ) There is a contingent truth not provoking any curiosity

Reason for saying it is a counterexample:

$$\forall\psi((\text{not-}[\psi \vdash \bot] \wedge \text{not-}[\vdash\psi]) \rightarrow \Diamond W\psi), \gamma \vdash \bot$$

Restricted Thesis in response to putative counterexample:

$$\forall\psi((\text{not-}[\psi \vdash \bot] \wedge \text{not-}[\vdash\psi] \wedge \text{not-}[W\psi \vdash \bot]) \rightarrow \Diamond W\psi)$$

The Restricted Thesis about curiosity (wondering whether) is substantive, informative and important. The objection that the restriction invoked is *ad hoc* is groundless.

Note that W is not even factive. That is, we do not have the inference '$W\varphi$; therefore φ'. This should have provided the clue to answer the question, raised by Hand and Kvanvig,[11] as to why I did not follow Fitch's example and avail myself of the factive inference '$K\varphi$; therefore φ' when proving the Fitch paradox. The answer is that the paradox does not rest on the factiveness of the epistemic operator K. Rather, it rests on logical features that it shares with the non-factive operator W, at least in so far as one is concerned with the inferential norms of ideally rational agents. Thus if φ is logically inconsistent, it would be inconsistent for an ideally rational agent to believe that φ. It would also be inconsistent for such an agent to wonder whether φ. Yet the would-be 'factive' inferences '$B\varphi$, therefore φ' and '$W\varphi$, therefore φ' are fallacious. It is of interest, therefore, that one can reproduce Fitch's result *without* using any rules that presuppose the factiveness of the operator K. This explains the reason why I 'prefer[red] this way of doing things.'

A certain sort of reflexive compounding of these operators with negation will lead to inconsistency. That is a feature of the logical structure of the propositional attitudes, and not a realism-relevant feature of the relationship between knowledge and truth. Thus, the kind of restriction that is clearly admissible in the case of the principle governing 'wonders whether' will be admissible (and non-*ad hoc*) in the case of the knowability principle.

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[11] Loc. cit., p. 422, n. 4.