Truth in a model

by

Neil Tennant[*]

Department of Philosophy
The Ohio State University
Columbus, Ohio 43210
email tennant.9@osu.edu

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The *extra-logical vocabulary* to be interpreted consists of a set of names, a set of function signs and a set of predicates. These sets are countable, and pairwise disjoint. A *model* M interpreting the extra-logical vocabulary has the following ingredients:

1. a domain D_M of individuals (henceforth: D);
2. an assignment, to each name a, of an individual a_M in D, as its bearer
3. an assignment, to each n-place function sign f, of a mapping $f_M : D^n \mapsto D$ as f's extension-in-M;
4. an assignment, to each n-place predicate letter P, of a subset P_M of D^n as P's extension-in-M.

Given (4), a primitive relational (as opposed to functional) M-fact is a matter of an n-tuple $<\alpha_1, \ldots, \alpha_n>$ lying in the extension P_M (where $\alpha_1, \ldots, \alpha_n$ are in D). It would be more natural to take P_M to be a translation, into the M-describing metalanguage, of the object-linguistic predicate P, so that such a fact consists, more simply, in $P_M(\alpha_1, \ldots, \alpha_n)$ being the case.

We shall denote by μ any assignment of individuals in D to (finitely many) individual variables. (The reason why we only need to deal with finitely many variables at a time is that every formula has at most finitely

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many free variables.) Given such an assignment μ, and given any individual α in D, we define $\mu(x/\alpha)$ to be the assignment that results from μ by extending it or modifying it so as to assign α to x. The empty assignment is denoted by $\emptyset$.

We now define the notion

$$M_\mu(t),$$

to be read as 'the M-relative denotation of the term t'.

Definition 1 (M_μ-relative denotations of terms)

1. *If x is an individual variable, then*

$$M_\mu(x) = \mu(x).$$

2. *If a is a name, then*

$$M_\mu(a) = a_M.$$

3. *If f is an n-place function sign, and $t_1, \ldots, t_n$ are terms, then*

$$M_\mu(f(t_1, \ldots, t_n)) = f_M(M_\mu(t_1), \ldots, M_\mu(t_n)).$$

Next we define the notion

$$M_\mu \models \varphi,$$

to be read as 'the extended model M_μ makes true the formula φ'.

Definition 2 (M_μ-relative truth-conditions of formulae)
Let μ deal with all the free variables in φ. Then

1.	$M_\mu \models P(t_1, \ldots, t_n)$	$\Leftrightarrow$	$P_M(M_\mu(t_1), \ldots, M_\mu(t_n))$
2.	$M_\mu \models \neg\varphi$	$\Leftrightarrow$	not$[M_\mu \models \varphi]$
3.	$M_\mu \models (\varphi \wedge \psi)$	$\Leftrightarrow$	$M_\mu \models \varphi$ and $M_\mu \models \psi$
4.	$M_\mu \models (\varphi \vee \psi)$	$\Leftrightarrow$	$M_\mu \models \varphi$ or $M_\mu \models \psi$
5.	$M_\mu \models (\varphi \rightarrow \psi)$	$\Leftrightarrow$	$M_\mu \models \varphi$ only if $M_\mu \models \psi$
6.	$M_\mu \models \exists x\varphi$	$\Leftrightarrow$	for some $\alpha \in D,\ M_{\mu(x/\alpha)} \models \varphi$
7.	$M_\mu \models \forall x\varphi$	$\Leftrightarrow$	for all $\alpha \in D,\ M_{\mu(x/\alpha)} \models \varphi$

Definition 3 (M-relative truth of a sentence)
Let φ be a sentence (i.e., a formula with no free variables). Then $M \models \varphi$ ('M makes φ true', or 'φ is true-in-M') $\Leftrightarrow M_\emptyset \models \varphi$.

Definition 4 *Let* Δ *be a set of sentences. Then* $M \models \Delta \Leftrightarrow$ *for every* φ *in* Δ*, we have* $M \models \varphi$*.*

Definition 5 $\Delta \models \varphi \Leftrightarrow$ *for every model* M *of the extra-logical vocabulary of* Δ, φ*: if* $M \models \Delta$*, then* $M \models \varphi$*.*