The temptation to use these notions of L-truth and L-falsity suggests another interesting point. If the objects of a theory could somehow be fixed a priori to be the numbers (as here) or physical objects or space-time points, we could get a different relation of "L-incompatibility" for which the underdetermination thesis might hold. Yet such a priori fixation of the objects of a theory is the epitome of what Quine maintains is impossible or meaningless. The primary thrust of his indeterminacy arguments[9] is that there is no fact of the matter, whether the objects of a theory are concrete things, their stages, their undetached parts, or even their Gödel numbers. Craig's example fails to support Quine's underdetermination thesis.

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THEORY CHANGE AND THE INDETERMINACY OF REFERENCE*

IN this paper I will argue that considerations about scientific revolutions show that many scientific terms are *referentially indeterminate*—there is no fact of the matter as to what they denote (if they are singular terms) or as to what their extension is (if they are general terms). In the opening section I will try to establish this general point of view by considering a particular scientific revolution, that in which Newtonian mechanics was replaced by the special theory of relativity. I will argue that this revolution reveals that the word 'mass' as used before relativity theory was discovered had no determinate denotation. Later on in the paper I will suggest that this indeterminacy in the terms of earlier scientists is evidence for an indeterminacy in many of our own scientific terms.

In addition to establishing the existence of indeterminacy, I want to show its import for semantic theory. There is a long tradition in semantics according to which the truth value of what someone says is determined by certain semantic features of the words (or word-tokens) he uses in saying it. Thus the truth value of an utterance of 'John loves Mary and Mary loves Bob' is determined by the denotations of the utterer's tokens of 'John', 'Mary', and 'Bob' (i.e., by the people the utterer referred to in using these names) and by the ex-

[9] Especially in *Ontological Relativity and Other Essays* (New York: Columbia, 1969).

* I am grateful to Michael Friedman, Clark Glymour, and David Hills, for helpful suggestions about this paper. My work on the paper was supported in its final stages by the National Endowment for the Humanities.

tension of his token of 'loved': the utterance is true if and only if the extension of 'loved' contains both the ordered pair whose first member is the denotation of (the token of) 'John' and that whose second member is the denotation of 'Mary', and the ordered pair whose first member is the denotation of 'Mary' and whose second member is the denotation of 'Bob'.[1] It has seemed to many philosophers that it is only because there are linguistic rules for determining the truth value of a sentence from the denotations of its component names, extensions of its component predicates, etc., that the notions of truth and falsity even make sense.

This point of view (I'll call it referential semantics) was adhered to by Frege, Carnap, and Tarski,[2] and has become increasingly common in recent years. But referential indeterminacy creates a serious problem for referential semantics. For we'll see that there are sentences with perfectly determinate truth values which contain referentially indeterminate names and predicates, so that it makes perfectly good sense to ask whether the sentence is true or false even though it doesn't make sense to ask what the name really denotes or what the real extension of the predicate is. Clearly, then, the fact that it makes sense to speak of the truth and falsity of such sentences cannot be based on the existence of linguistic rules that determine truth value from denotation and extension.

So if my argument for referential indeterminacy works, it follows that we have to revise the program of referential semantics to some extent. The second part of the paper will be concerned with the question of what sort of revision is required in order to handle the sort of indeterminacy whose existence is revealed by scientific revolutions.

I

Before Einstein's special theory of relativity, physicists accepted a great many assertions involving the term 'mass' that are no longer accepted today. For instance, Newton and his successors accepted the following claims:

(1) The mass of a particle is equal to twice its kinetic energy divided by the square of its velocity.

[1] It would be possible to formulate referential semantics in terms of properties rather than sets: we could say that the utterance 'John loves Mary' is true if and only if the person denoted by 'John' bears the relation (2-place property) denoted by 'loves' to the person denoted by 'Mary'. I believe that if properties are carefully distinguished from meanings, there are certain advantages to proceeding in this manner; but for our present concerns it is irrelevant whether we choose sets or properties, so I have stuck to the more common policy of relying on sets.

[2] This is more true of what Tarski *ought* to have said (given his logical discoveries) than of what he *did* say: cf. my paper "Tarski's Theory of Truth," this JOURNAL, LXIX, 13 (July 13, 1972): 347–375.

and

(2) Mass is conserved in all interactions.

Part of the novelty of Einstein's theory was that (1) and (2) and most of Newton's other assertions involving 'mass' were given up. This fact has led some philosophers (e.g., Thomas Kuhn[3]) to claim that before relativity theory was discovered the term 'mass' did not denote (or refer to)[4] the same physical quantity[5] it denotes (or refers to) today. These philosophers, in other words, *deny* the following claim:

(3) Newton's term 'mass' denoted *mass*; i.e., Newton was referring to the same physical quantity that *we* refer to when *we* use the word 'mass'.

It seems to me very unlikely that the falsity of (3) can be established solely on the sorts of grounds that Kuhn utilizes, viz., on the fact that Newton had many beliefs involving 'mass' that are no longer accepted: there is nothing at all incoherent in the position that Newton was referring to mass even though he had a great many false beliefs about it (cf. note 3). Moreover, we should note that the false beliefs about mass that Newton must have had for (3) to be right are all *approximately* true—e.g., we can approximate (1) by the weaker claim

(1*) At low velocities, the mass of a particle is almost precisely equal to twice its kinetic energy divided by the square of its velocity.

[3] *The Structure of Scientific Revolutions* (Chicago: University Press, 1962), pp. 100–101. A similar conclusion is suggested by a passage from Quine's *Word and Object* (Cambridge, Mass.: MIT Press, 1960), p. 16, about the term 'neutrino'. Judging from this passage we might expect that Quine would argue as follows:

> Since Newton's and Einstein's theories differed so much, they cannot have been referring to the same quantity. The fact that both physicists used the word 'mass' is not significant. To discern two phases here, the first an agreement as to what physical quantity is involved (viz., mass) and the second a disagreement as to what it's like (what laws it obeys), is absurd.

But this would be an *ignoratio elenchi*: the issue is not whether Newton "knew what mass is" or whether he and Einstein "agreed as to what mass is," but whether he *referred* to mass and whether he and Einstein referred to the same thing. Even if one is not inclined to say that Newton "knew what mass is," it is still possible to say that he referred to mass, and saying so might be important in enabling you to characterize the truth value of his assertions in terms of referents of the component terms.

[4] I use these terms more or less interchangeably. (Usually I say that a *person* refers, and that the *word-token* he uses in referring to x denotes x.)

[5] A physical quantity can be viewed as a function that assigns a value to each physical object (or measurable set of spatial points). A quantity-term like 'mass' can take the role of a function symbol, as in (1) above, or it can take the role of a singular term, as in the abbreviated form of (1): 'Mass equals twice kinetic energy divided by velocity'. In *either* case I say that it denotes the physical quantity.

and this approximating claim is true according to relativity theory. And there is *certainly* nothing incoherent in the position that Newton was referring to mass even though he had a great many false-but-approximately-true beliefs about it; indeed, the fact that most of Newton's beliefs involving 'mass' come out approximately-true-though-strictly-false if we regard them as being about mass looks like evidence *for* (3) rather than *against* it.

Nevertheless, there are *other* considerations about Newtonian physics and its relation to special relativity which *do* call (3) into question: these considerations show, I contend, that it simply doesn't make sense to ask what physical quantity Newton and other pre-relativity physicists referred to when they used the term 'mass'.

If one opens a textbook on special relativity, one may find assertions such as the following:

(R) The mass of a particle is equal to the total energy of the particle divided by the square of the speed of light. Since the total energy of a particle with respect to one frame of reference differs from the total energy with respect to other frames of reference, but the speed of light is the same in all frames of reference, it follows that the mass of the particle has different values in different frames.

That's what one will find in *some* textbooks, but in others one will find something quite different:

(P) The mass of a particle is equal to the *nonkinetic* energy of the particle divided by the square of the speed of light. Since the nonkinetic energy of a particle is the same in all frames of reference, and a similar claim holds for the speed of light, it follows that the mass of the particle has the same value in all frames.

(R) and (P) appear to conflict with each other; but is this conflict genuine? A closer examination of the two textbooks will reveal that both agree that the total energy divided by c^2 (where c is the speed of light) has a different value in different frames, and that the nonkinetic energy divided by c^2 has the same value in all frames: perhaps, then, the "disagreement" is simply over which of these quantities is to be called "mass." This hypothesis is confirmed by the fact that the textbooks agree about the ratio of these quantities: they agree that

$$\frac{\text{nonkinetic energy}/c^2}{\text{total energy}/c^2} = \sqrt{1 - \frac{v^2}{c^2}}$$

(where v is the speed of the particle). In fact it turns out that if you take any formula of Textbook P (the textbook from which (P) was extracted) and replace 'mass' everywhere by 'mass times $\sqrt{1 - (v^2/c^2)}$, you get a formula that is acceptable according to Textbook R, and if you take any formula of Textbook R and replace 'mass' everywhere by 'mass divided by $\sqrt{1-(v^2/c^2)}$, you get a formula that is acceptable according to Textbook P. It seems reasonable to say, then, that there are two physical quantities involved here: Textbook R uses the term 'mass' for a quantity that can be called *relativistic mass*, which is equal to total energy/c^2; and Textbook P uses the term 'mass' for a different quantity that can be called *proper mass*, which is equal to nonkinetic energy/c^2. The appearance of conflict between the two textbooks arises solely from the fact that one uses the word 'mass' for one quantity and the other uses the same word for a different quantity.

With these facts in mind, let us return to the question of what physical quantity Newton was referring to when he used the term 'mass'. It is natural to imagine Physicist R making the following claim:

(3R) Newton's term 'mass' denoted *mass*; i.e., it denoted total energy/c^2.

But it is equally natural to imagine Physicist P claiming

(3P) Newton's term 'mass' denoted *mass*; i.e., it denoted nonkinetic energy/c^2.

Putting their claims into unambiguous terminology, we get

(HR) Newton's word 'mass' denoted relativistic mass.

and

(HP) Newton's word 'mass' denoted proper mass.

How are we to decide between these claims?

My claim is that we *can't*. Before relativity theory was discovered (I will argue), the word 'mass' was *referentially indeterminate*: it did not *lack* denotation, in any straightforward sense; on the contrary, there are *two* physical quantities that each satisfy the normal criteria for being the denotation of the term. To put the point more precisely, let a *positive analytical hypothesis* for Newton's use of the term 'mass' be a sentence of the form

Newton's word 'mass' denoted *X*.

where '*X*' is the name of a physical quantity, and let the *negative analytical hypothesis* be the sentence

(HΛ) Newton's word 'mass' denoted nothing whatsoever.

I claim that the negative analytical hypothesis must be rejected, and so must all positive hypotheses except for (HP) and (HR). I also claim that (HP) and (HR) are each extremely plausible (or more properly, that each would be extremely plausible were it not for the existence of the other), and that *there is no basis for choosing between them.* It isn't merely that we don't *know* whether Newton was referring to proper mass or to relativistic mass; I claim that *there is no fact of the matter* as to which of these quantities he was referring to.

The first point I will argue is that there is no basis for choosing *between* (HR) and HP), i.e., for accepting one of these hypotheses and rejecting the other. In order to appreciate the difficulty of choosing between them, it is necessary to know two central tenets of Newtonian mechanics:

(4R) Momentum = (mass) v

and

(5P) For any two frames of reference, mass with respect to frame 2 = mass with respect to frame 1.

The first of these was formulated quite explicitly in every presentation of Newton's theory, often as the "definition" of momentum; the second was considered so obvious that it never occurred to anyone to state it, but certainly an advocate of the theory would have assented to it had anyone thought to ask him, and its truth was presupposed in all of the theory and practice of Newtonian mechanics. Now, were these two tenets of Newtonian mechanics correct? They can't *both* have been correct; for, according to relativity theory, the momentum of a particle divided by its velocity has different values in different frames of reference, whereas (4R) and (5P) together entail that the momentum divided by the velocity has the same value in different frames.

Relativity theory, then, shows that the *conjunction* of the Newtonian tenets (4R) and (5P) was false; and so it would be natural to expect that it would also show that a particular one of these tenets was false. Unfortunately, this natural expectation seems to be wrong. For the fact is that momentum *does* equal *relativistic* mass times velocity, and does *not* equal *proper* mass times velocity; whereas (as we've seen) *relativistic* mass does *not* have the same value in all frames, whereas *proper* mass *does*. In other words, *some* physicists today use the word 'mass' in such a way that their tokens of (4R) are true and their tokens of (5P) false, and if Newton was using the word in the way that they do (i.e., for relativistic mass), then his tenet (4R) was true and his tenet (5P) was false. But *other* physicists use the word 'mass' in such a way that their tokens of (4R) are false and their tokens of (5P) are true, and if Newton used the word 'mass' in the way that *they* use the word (i.e., for proper mass), then his tenet (4R) was false and his tenet (5P) was true. I contend that there is no basis whatever for deciding *between* these two possibilities. *The* conjunction *of Newton's tenets (4R) and (5P) was objectively false, but there is no fact of the matter as to which of the conjuncts was true and which false, and hence no fact of the matter as to whether the word 'mass' as it occurred in them denoted relativistic mass or proper mass.*

The difficulty of choosing between (HR) and (HP) arises, then, from the fact that (4R) and (5P) were both extremely central to Newton's theorizing and to his scientific practice; and I can see no basis for asserting that a particular one of them was more central than the other. Perhaps, however, we could decide between (HR) and (HP) on the basis of other tenets of Newtonian physics? I don't believe so. Most of the *theoretical* claims of Newtonian physics, like (1) and (2), come out false (though *approximately* true) whether we take Newton as referring to relativistic mass or to proper mass; and most of Newton's *experimental* claims, like

(6) The mass of Object *A* is between 1.21 and 1.22 kilograms [said after putting Object A onto a pan balance and accurately weighing it].[6]

come out true either way.

Another way we might try to choose between (HR) and (HP) is to look, not at Newtonian mechanics, but at special relativity; for if one were to find that one of the quantities played a much more important role in special relativity than the other one played, some "principle of charity" might provide some basis for asserting that the more important quantity was the one that Newton was really talking about. Unfortunately, however, proper mass and relativistic mass play about equally important roles in special relativity theory. *Some* laws of physics come out looking simpler in terms of relativistic mass than in terms of proper mass: thus if we let 'mass' stand for relativistic mass we get (4R), but if we let 'mass' stand for proper mass we get the more complicated formula:

[6] I use here the fact that Newton made experimental assertions only about objects that were moving slowly (in comparison with the speed of light), and for a slowly moving object the proper mass and relativistic mass are virtually identical. (It might be asked "Slowly moving with respect to what reference frame? No specific frame is mentioned in (6)." But no specific frame is mentioned in assertions like 'Object *A* is traveling at 10 ft/sec' either, and yet we often adjudicate the truth or falsity of such assertions. We do so by supposing that one frame of reference is *intended* by the speaker, and evaluating the truth or falsity of the utterance with respect to the intended frame. So the answer to the question is: slow-moving with respect to the intended frame.)

(4P) $\text{Momentum} = \frac{(\text{mass})\, v}{\sqrt{1 - (v^2/c^2)}}$

But *other* laws of physics come out simpler in terms of proper mass than in terms of relativistic mass: when 'mass' stands for proper mass we get the simple transformation law (5P), but when 'mass' stands for relativistic mass we get instead

(5R) Mass with respect to frame 2

$$= (\text{mass with respect to frame 1})\, \frac{\sqrt{1 - (v_1^2/c^2)}}{\sqrt{1 - (v_2^2/c^2)}}$$

(where v_1 and v_2 are the speeds of the particle in frame 1 and frame 2, respectively). Taken together, the laws of special relativity come out looking just about as simple when formulated in terms of one of these quantities as they look when formulated in terms of the other quantity. This explains why the division between those physicists who find it more aesthetically pleasing to formulate the theory in terms of relativistic mass and those who prefer to formulate it in terms of proper mass is just about 50/50.

I think that my last four paragraphs make it highly plausible that there is no basis for choosing between the hypotheses (HP) and (HR). But this contention is compatible with a number of positions which I now want to argue against. The first position is

(HN) Newton wasn't referring either to proper mass or to relativistic mass; instead he was referring to a quantity called "Newtonian mass" which had some of the properties of each.

(More precisely, Newtonian mass is equal to momentum divided by velocity—in this it is unlike proper mass but like relativistic mass. But, unlike relativistic mass, Newtonian mass has the same value in all reference frames. Newtonian mass also has some properties that distinguish it both from proper mass and from relativistic mass: e.g., it is equal to twice the kinetic energy divided by v^2, and it is conserved in all interactions.)

The view that what Newton was referring to was "Newtonian mass," as just characterized, seems to be advocated by Kuhn. In discussing Einstein's concepts of space, time, and mass, Kuhn writes:

> The *physical referents of* these Einsteinian concepts are by no means identical with those of the Newtonian concepts that bear the same name. (Newtonian mass is conserved; Einsteinian is convertible with energy . . .) [*op. cit.* p. 101, emphasis mine].[7]

[7] It is odd that Kuhn adopts this view. Only a few pages earlier (97) he had correctly asserted "Einstein's theory can be accepted only with the recognition that Newton's was wrong"; but if Newton was referring to "Newtonian mass" and Einstein to "Einsteinian mass," *both* theories would be right.

But this view seems wholly unreasonable, for what Einstein showed is that there *is* no such quantity as "Newtonian mass"; and unless one holds that the world used to obey Newton's laws but started obeying Einstein's laws one day, it is clear that there was no "Newtonian mass" in Newton's time either. I'm not denying that in Newton's time the word 'mass' *meant* something different than the word 'mass' (or its counterparts 'relativistic mass' and 'proper mass') means today; on certain construals of the term 'meaning', this seems to be perfectly true. What I'm denying is not a claim about *meaning*, but a claim about *reference* or denotation: I'm denying that there is or ever was such a quantity as "Newtonian mass," and hence I'm denying that Newton could have ever referred to "Newtonian mass" when he used the word 'mass'.

Once we've rejected all appeals to "Newtonian mass," it becomes clear that there are no positive analytical hypotheses that are more plausible than (HR) and (HP). But now we must examine the negative analytical hypothesis (HΛ), that Newton's word 'mass' was simply denotationless, in the way that 'Santa Claus' is denotationless. I claim that, if we regard Newton's tokens of 'mass' as simply denotationless, we are forced to assign the wrong truth values to Newton's sentence tokens.

In order to make the argument good, we have to know how a denotationless term like 'Santa Claus' affects the truth value of sentences that contain it. One view, sometimes attributed to Frege, is that all sentences containing denotationless terms lack truth value; another view is that sentences containing denotationless terms are all false. If either of these views were true, my argument would be simple: I could merely point out that Newton made many utterances with the term 'mass' that all current physicists would agree to, whichever way they used the term 'mass'. For instance, he doubtless made experimental assertions like (6), and he doubtless made theoretical assertions like

(7) To acclerate a body uniformly between any pair of different velocities, more force is required if the mass of the body is greater.

When Newton made such assertions as these, he would certainly seem to have been saying something true; and, since his tokens of (6) and (7) were true, the tokens of 'mass' that occur in them can't simply have lacked denotation in the way that 'Santa Claus' lacks denotation.

As it stands, the above argument will not do; for it depends on the view that a sentence containing a term like 'Santa Claus' is always truth-valueless or false, and I regard this view as incorrect. However something *like* that view is correct, and once we see what it is we can see how the above argument can be patched up so that it really does show that Newton's tokens of 'mass' were not denotationless.

Consider the following three sentences which contain the word 'Santa Claus':

(8) Santa Claus doesn't exist.
(9) Johnny saw Santa Claus today.
(10) Santa Claus had a wart on his left shoulder.

I would regard the first of these as true, the second as false, and the third as truth-valueless. Others may have different attitudes toward (9) or (10), but one thing will, I think, be generally agreed: if you take any other nondenoting singular term and substitute it for 'Santa Claus' in (8), (9), and (10), the resulting sentences will have the same truth value as the original, or have no truth value if the original lacked truth value. For instance, let's assume that 'Moses' doesn't denote anyone; then certainly the sentence

(8′) Moses doesn't exist.

is true. Moreover, anyone who regards (9) as false will also regard

(9′) Johnny saw Moses today.

as false, and anyone who regards (9) as truth-valueless will regard (9′) as truth-valueless. Similarly,

(10′) Moses had a wart on his left shoulder.

will appear truth-valueless to those who regard (10) as truth-valueess, and false to those who regard (10) as false. These examples suggest that Leibniz's well-known principle

(LL) If two terms each denote the same object, then substitution of one term for the other (in nonquotational, nonintentional, etc., contexts) always preserves truth value.

has a counterpart for nondenoting names:

(LL*) If two terms each denote nothing whatsoever, then substitution of one term for the other (in nonquotational, nonintentional, etc., contexts) always preserves truth value (or lack of truth value).

I think that (LL*) accords with nearly everyone's pretheoretic views as to the truth values of sentences containing terms like 'Santa Claus' and 'Moses.'[8] It is fortunate that this is so, for (LL*) is just

[8] It might be objected that 'Santa Claus flies reindeer' is true, whereas 'Moses flies reindeer' is not. But it seems to me that 'Santa Claus flies reindeer' is true only when it is elliptical for 'The story says that Santa Claus flies reindeer'; in that sentence, 'Santa Claus' occurs in an intentional context and so is immune to (LL*).

as important a principle for the semantics of nondenoting terms as (LL) is for the semantics of denoting terms. If two names each have no denotation whatsoever, they are completely alike from a denotational point of view; so how *could* substitution of one for the other affect truth value? The answer that will be given by any advocate of referential semantics is that it couldn't: if substitution of one term for another affects truth value, these terms must be different from each other, denotationally speaking, and that means that they are *not both* simply denotationless.

Using this principle, we can show that the term 'mass' (as used by Newton) was not denotationless. For suppose we replace the word 'mass' in (6) and (7) by the denotationless term 'phlogiston', getting

(6′) The phlogiston in Object *A* is between 1.21 and 1.22 kilograms.

and

(7′) To accelerate a body uniformly between any pair of different velocities, more force is required if the phlogiston in the body is greater.

Clearly no one will regard (6′) or (7′) as true; so if 'mass' were denotationless, then (6) and (7) wouldn't be true either. We're talking, of course, about (6) and (7) as used by Newton: the analytical hypothesis (HΛ) does not commit us to the claim that when (7) is used *today* it is untrue, but only to the claim that when (7) was used by *Newton* it was untrue. But even that seems bad enough—since all relativity theorists (whether they use 'mass' for proper mass or for relativistic mass) agree to the truth of (7), it seems grossly uncharitable to deny that Newton can have said something true by uttering (7). Similarly, it seems uncharitable to deny that when Newton referred to a slow-moving object whose proper mass and relativistic mass were nearly identical and were between 1.21 and 1.22 kilograms, he could have said something true by uttering (6). By calling Newton's utterances of (6) and (7) false, (HΛ) conflicts with the principle of charity; in this respect it is vastly[9] inferior both to (HR) and to (HP).[10]

[9] Note that (HΛ) is incompatible even with the view that (6) and (7) are *approximately* true; for any approximating claim would also come out untrue if evaluated according to (HΛ). Similarly, (1) and (2) come out approximately true according to (HP) and (HR), but not according to (HΛ). (These remarks hold not only on the conception of approximate truth introduced earlier, but on any other conception in which the approximate truth of a sentence depends on the denotations and extensions of its parts.)

[10] I would prefer not to have to rest my argument on the appeal to charity: I would prefer it if I could show that the purpose for which we want the notion of truth would not be satisfied if we declared (6) and (7) false in these circumstances.

We now have all the components of the argument for the referential indeterminacy of Newton's word 'mass'. We've seen that, of all the positive and negative analytical hypotheses, the only two with any plausibility are (HP) and (HR). Each of these two hypotheses would be plausible, were it not for the existence of the other, but we have found no basis whatever for choosing between the two. It could be contended, I suppose, that our inability to choose between (HP) and (HR) is due simply to ignorance—that one of (HP) and (HR) is correct and the other incorrect, and the only trouble is that we don't know which. But the only way to give this contention any plausibility is to state what sort of information is likely to decide between the two possible denotations; and the only prima facie likely bases of choice which I can think of have been ruled out earlier in the paper.

It seems implausible, then, that one of (HP) and (HR) is correct and the other incorrect: the situation is not that we don't *know* what Newton's word denoted, but that Newton's word was referentially indeterminate. But can this conclusion be tolerated? Surely (HP) and (HR) can't *both* be correct, and so if it is not the case that one is correct and the other incorrect, it follows that *neither* (HP) nor (HR) is correct. Yet we have also rejected all the obvious *alternatives* to (HP) and (HR) (i.e., the other positive analytical hypotheses, and the negative one), and this seems to show that there is simply no coherent way of using the term 'refers' in connection with Newton's word 'mass'. In spite of this, there are many of Newton's utterances containing the word 'mass' that we want to regard as true—(7) was one example—and there are many also that we want to regard as false, e.g., the conjunction of (4R) and (5P). It follows, then, that the truth and falsity of these utterances simply cannot be explained on the basis of what Newton was referring to when he used the word 'mass', for there is no coherent way of explaining what he was referring to. In other words, indeterminacy rules out the possibility of referential semantics.

II

In *Word and Object* Quine wrote

> If there were . . . an unknown ["ideal theory" θ (say a limit that would be attained by applying scientific method forever) which we could regard as completely true, *still* this would not settle the truth-

I don't know in detail how to show this, but I suspect that by further developing the remarks on the purpose of the notion of truth in the last section of my Tarski paper, *op. cit.* (and also arguing that we have good reason to suspect that many of our *own* words will someday be in the position that Newton's word 'mass' is in today), we could get the desired proof.

> value of] actual single sentences. We could not say . . . that any single sentence S is true if it or a translation belongs to θ, for there is in general no sense in equating a sentence of a theory θ with a sentence S given apart from θ. Unless pretty firmly and directly conditioned to sensory stimulation, a sentence S is meaningless except relative to its own theory; meaningless intertheoretically (23/4).

It is inessential to Quine's claim that θ be taken as a "limit theory"; we could equally well take θ to be current scientific theory, which is treated in most contexts *as if* it were completely true. Quine's point then, in the first part of the passage, is that there are sentences S from theories *not* now accepted such that (i) we can not equate them with any sentences of current scientific theory and (ii) we can not find any objective grounds for deciding whether they are true of false. The preceding section provides a confirmation of this part of Quine's claim: both (4R) and (5P) are examples of such an S. In other words, the observations of the preceding section show not only a failure of the concept of reference to do what it was supposed to do (viz., explain truth), but also a failure of the concept of truth at least as applied to certain *single sentences* of a theory: Newtonian physics *as a whole* is objectively false, but there is no fact of the matter as to how the falsity of the theory as a whole is to be distributed among the individual sentences of the theory.

This much I agree,[11] but still there is part of Quine's claim that I do not accept (or even fully understand): the part that concludes that S is "meaningless intertheoretically." In the rest of this paper I will sketch a way to handle referential indeterminacy (of the sort we've been discussing) which does not accord with this last Quinean claim.

My approach will be to try to preserve as much of referential semantics, and of the claim (3), as *can* be preserved in the face of indeterminacy. The argument of the last paragraph of the preceding section shows, I believe, that there is at present no coherent way of using the term 'denotes' in connection with Newton's word 'mass'. I think that in this situation we have to develop a new semantic terminology that is capable of handling referentially indeterminate expressions. The terminology I propose is 'partially denotes': I want to say that Newton's word 'mass' partially denoted proper mass and partially denoted relativistic mass; since it *partially* denoted *each* of them, it didn't *fully* (or *determinately*) denote *either*.

The novel feature of partial denotation is that a singular term can

[11] Except that the last sentence holds only if we restrict our consideration to methods of distribution that accord with classical 2-valued semantics. This will become clearer shortly.

partially denote more than one thing, and a quantity term like 'mass' can *partially* denote more than one quantity. A term that partially denotes more than one quantity is called *referentially indeterminate*[12]; a term that partially denotes exactly one quantity is said to *fully* (or *determinately*) *denote* that quantity; and a term that doesn't partially denote anything is said to be *denotationless*. I do not try to define 'denotes' in terms of 'partially denotes', for I don't think that this can be done in any acceptable way. (Probably the *best* such definition is to take 'denotes' as equivalent to 'fully denotes'; but this would have the unattractive feature that indeterminate terms would not denote anything and yet would not be denotationless.) I find it more natural simply to abandon the term 'denotes', except when dealing with terms that are perfectly determinate (or can be assumed, in a given context, to be perfectly determinate).[13]

If the term 'denotes' is abandoned, we can no longer express (HP), (HR), and (HΛ). But of course we can express close analogues of them, viz.,

(HP*) Newton's word 'mass' fully denoted proper mass; i.e., it partially denoted proper mass and nothing else.
(HR*) Newton's word 'mass' fully denoted relativistic mass; i.e., it partially denoted relativistic mass and nothing else.
(HΛ*) Newton's word 'mass' was denotationless; i.e., it didn't partially denote anything at all.

But none of these analytical hypotheses are acceptable, for precisely the same reasons that none of (HP), (HR), and (HΛ) were acceptable. The problem with (HP*) and (HR*) is that each of them discriminates between proper mass and relativistic mass, and there ap-

[12] This definition and that which follows are intended to apply to term *tokens*, not to term *types*. This is necessary in order to distinguish indeterminacy from ambiguity. A term is *ambiguous* if different tokens of it have different semantic features; thus 'mass' as used today is ambiguous, since physicist *R*'s tokens denote relativistic mass while physicist *P*'s tokens denote proper mass. There is thus a sense in which the term *type* 'mass' as used today denotes (and hence partially denotes) more than one thing; but this does not demonstrate the existence of indeterminacy or show the failure of referential semantics; it merely shows that referential semantics has to be applied to tokens instead of to types. For a case of indeterminacy one needs to look to pre-relativity physics: here each *token* of 'mass' partially denoted two different quantities.

[13] I would defend this last approach on the grounds that the word 'denote' is itself referentially indeterminate: it partially signifies the relation of full denotation and partially signifies the relation of partial denotation. For referentially determinate terms, full denotation and partial denotation coincide, and so we can safely apply the word 'denotes' to these terms without having to take the indeterminacy of 'denotes' into account. But if a term t partially denotes an object x without fully denoting it, the statement that t denotes x will lack truth value; and so to avoid the complication of truth-valueless metalinguistic sentences, it is best to drop the indeterminate term 'denotes'.

pears to be no basis for such discrimination. In this respect, (HΛ*) is better than (HP*) and (HR*) ,but only at a high cost: according to (HΛ*), Newton's word 'mass' bears no semantic relation to anything at all, and as a consequence 'mass' turns out to be just like 'phlogiston', denotationally speaking. These objections to (HP*), (HR*) and (HΛ*) are not shared by the new analytical hypothesis

(HI) Newton's word 'mass' partially denoted proper mass and partially denoted relativistic mass and didn't partially denote anything else.

In particular, (HI) rules that 'mass' and 'phlogiston' are not alike, denotationally speaking: for clearly 'phlogiston' either does not partially denote anything, or else partially denotes a whole bunch of things, none of which is either form of mass.

If we accept the idea that Newton was partially referring to proper mass and partially referring to relativistic mass when he used the term 'mass', which of these partial denotations are we to say affected the truth values of Newton's utterances? The answer of course is "both." When Newton uttered the sentence

(5P*) Mass is independent of the frame of reference.

did he say something true? The relevant facts, you'll recall, are that relativistic mass is not independent of the frame of reference, but proper mass is; so the answer is that when Newton uttered this sentence what he said was sort of true and sort of false—it was imprecise, and the imprecision mattered. There are also cases where the imprecision does not matter: (7) was such an example, since both the modern analogs of (7) are true.

In order to turn these observations into an adequate truth definition for sentences containing indeterminate expressions, we have to extend our analysis of indeterminacy beyond singular terms and quantity terms, to other parts of speech such as general terms, for it is plausible that indeterminacy can arise in them also. An example, I suspect, is the word 'gene' as used by all biologists until a very few years ago: one analytical hypothesis governing the use of this term by earlier biologists is that it had the set of cistrons as its extension, a second is that it had the set of mutons as its extension, and a third is that it had the set of recons as its extension.[14] There is also a negative analytical hypothesis, that the extension of 'gene' was the empty set.[15] But I think it could be argued, as was argued before

[14] Cf. Kenneth Schaffner, "Approaches to Reduction," *Philosophy of Science* XXXIV, 2 (June 1967): 137–147, pp. 142/3; and references cited there.

[15] For predicates actually, there are *two* possible "negative analytical hypotheses": (a) that the predicate has the empty set as its extension, and (b)

with the term 'mass', that the negative hypothesis is inadequate, and that all the positive hypotheses are inadequate too, since they all ascribe to the word 'gene' (as used by earlier biologists) a determinate extension when the word was in fact indeterminate. To handle such cases of indeterminacy, I introduce the term *partial extension*; this allows us to express analytical hypotheses such as

> The word 'gene' had the set of cistrons as one partial extension, the set of mutons as another, and the set of recons as a third.

It is convenient also to introduce the words *signify* and *partially signify*, by saying that a determinate predicate signifies its extension and that an indeterminate predicate partially signifies each of its partial extensions.

In order to give a semantics for indeterminate expressions, let's introduce the term 'structure'; a *structure* for a sentence is a function that maps each name or quantity term of the sentence into some object or quantity, and maps each predicate into some set. The structure *m corresponds* to the sentence if each name or quantiy term of the sentence partially denotes the thing that *m* assigns to it, and each predicate partially signifies the set that *m* assigns to it. Now, for each structure *m*, we can apply the standard referential (Tarski-type) semantics to determine whether the sentence is *m-true* or *m-false*, i.e., true or false *relative to m*. (To say that the sentence is *m*-true is to say that it *would* be true if the denotations and extensions of its terms were as specified by *m*.) We can then say that a sentence is *true* (false) if it is *m*-true (*m*-false) for every structure *m* that corresponds to it. Putting all these definitions together, we get definitions of truth and falsity in terms of partial denotation and partial signification.

A few examples will illustrate how the truth definition works and show the plausibility of it. Consider first the sentence (5P*), and for the sake of illustration regard everything after the 'mass' as a primitive predicate. Then there are two structures corresponding to this sentence: both assign to the predicate 'is-independent-of-the-frame-of-reference' the same set (viz., the set of frame-independent quantities), but one (m_1) assigns proper mass to 'mass', whereas the other (m_2) assigns relativistic mass to 'mass'. Relative to m_1 the sentence is true, and relative to m_2 it is false, and, since these relative truth

that it doesn't have an extension at all. (b) sounds a bit more reasonable than (a), but only because (b) is compatible with the hypothesis of indeterminacy set out later in this paragraph; (b) is not, in other words, a viable *alternative* to the hypothesis of indeterminacy, any more than (a) is, for taken by itself (i.e., without use of the concept of partial extension) (b) would obliterate the semantic differences between any two referentially indeterminate predicates.

values differ, the sentence is not determinately true or false. A similar analysis will show that (4R) had no determinate truth value for Newton, but that (6) and (7) were determinately true.

Now let's consider sentences in which the same indeterminate term occurs twice. An interesting example is the conjunction of (4R) and (5P). Again there are two structures corresponding to this sentence. Relative to the one that assigns proper mass to 'mass', the first conjunct is false and the second true, so that the whole sentence is false; relative to the other structure the first conjunct is true and the second false, so that again their conjunction is false. *The truth definition tells us, then, that the conjunction of (4R) and (5P) was false —even though each conjunct lacked truth value (so that there is no fact of the matter as to* which *conjunct was false)*. This is precisely the conclusion that seemed desirable earlier.

Finally, we have to consider the very important case where there are two different indeterminate expressions in the same sentence. One such example of double indeterminacy is

(11) The mass of a gene is less than a microgram.

(as used by pre-1950 geneticists). Here the semantics says, plausibly enough, that the utterance was true, since for cistrons, recons, and mutons, both the proper mass and relativistic mass are less than a microgram. But it would be well to consider also a case where a sentence contains two indeterminate terms from the same theory. In order to get such an example, let us suppose not only that was Newton's word 'mass' indeterminate, but that his word 'velocity' was also indeterminate—it was indeterminate between rate of change of distance with respect to normal time t, and rate of change of distance with respect to proper time T. (Perhaps this is implausible, but it illustrates the semantics admirably.) Now let us reconsider sentence (4R) on this supposition. We find that, since $dT = dt\sqrt{1 - (v^2/c^2)}$,

momentum equals (relativistic mass) dx/dt
momentum does not equal (relativistic mass) dx/dT
momentum does not equal (proper mass) dx/dt
momentum equals (proper mass) dx/dT.

As a result, (4R) comes out m-true for two of the structures m that correspond to it and m-false for the other two; therefore it comes out truth-valueless, just as it did before. It seems to me that this is just what we would intuitively want to say if we believed that 'mass' and 'velocity' were both indeterminate in the way I've supposed.[16]

[16] This may perhaps be controversial: some might want to say that Newton's utterances of (4R) were true, and that only the first and last of the four facts

I think these examples make it plausible that the truth definition I've given, together with the appropriate hypotheses about what things the earlier theorist was partially referring to or partially signifying, leads to the sort of results we want. If so, then I have established the utility of the concept of partial denotation in dealing with theory change, and shown that scientific revolution can affect the semantic features of scientific terms in more complicated ways than many philosophers have allowed. No longer are we confined to the three alternatives: (i) term *t* has switched denotations during revolution *r* [as (HN) held]; (ii) *t* has acquired a denotation during *r* [as (HΛ) held]; and (iii) *t* has kept the same denotation through *r*. Now there are other possibilities, most notably (iv) *t* has undergone a *denotational refinement* during *r*; i.e., the set of things that it partially denoted after *r* is a proper subset of the set of things it partially denoted before. This added possibility is of considerable importance for debates about "incommensurability" in science: it shows that we can accept the claim that we can't always *equate* a term from one theory with a term from a later theory, and still deny the incommensurability thesis, i.e., the thesis that the earlier and later terms cannot objectively be compared with respect to referential properties.

It is worth making two final points about denotational refinement. First, the 'mass' example is not an example of denotational refinement pure and simple: it is an example of *double refinement*, since some physicists have refined 'mass' into a word for relativistic mass while others have refined it into a word for proper mass. I hope that by concentrating on this example I have not obscured the fact that simple refinement (where the term is refined in only one direction) is also a possibility. That this really is a possibility should be obvious: 'mass' would have undergone a simple refinement rather than a double refinement if everyone had followed Einstein's example in adhering to (4R) and rejecting (5P). Simple refinement will doubtless be less obtrusive than multiple refinement (since there will be no linguistic disagreement resulting from the revolution like the lin-

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listed above *ought* to be considered relevant to the truth value of these utterances. In other words, it might be held that only *certain* combinations of the partial denotations and partial extensions of the terms ought to affect the truth value of the whole sentences and that therefore my semantics needs to be complicated by imposing some restrictions on which combinations are considered.

I don't object to this contention in principle, but I also don't see any very good reason for supposing it to be true (for the kind of indeterminacy under consideration in this paper). There may be *other* sorts of indeterminacy for which the more complicated semantics is necessary, but I will defer consideration of that more complicated kind of indeterminacy (*correlative indeterminacy*, I call it) to a later paper.

guistic disagreement between Physicists R and P); but it seems plausible that it does sometimes occur.[17]

Second point: if I am right in thinking that denotational refinement is a fairly common feature of scientific revolutions, that suggests that future scientists may very well refine many of our current scientific terms, and hence that many of our current scientific terms are referentially indeterminate. (In fact, induction from the indeterminacy of terms in earlier theories may even suggest that science will *never* reach the stage where all of its terms are perfectly determinate.) I don't think that this conclusion conflicts with anything that is reasonably called "scientific realism"; but an adequate discussion of this matter is beyond the scope of the present paper. I will conclude the paper by returning briefly to the comparison with Quine's views begun earlier in this section.

III

By modifying the program of referential semantics in the way I have suggested, we come to rather different conclusions about indeterminacy from those reached by Quine. Quine thinks the existence of indeterminacy shows that scientific terms are "meaningless [and denotationless] except relative to [their] own theory; meaningless [and denotationless] intertheoretically." This claim is then used to buttress his tenet that "truth . . . is immanent [i.e., relative] to the conceptual scheme"[18]: for the natural way to get an objective (nonrelative) notion of truth is to try to explicate that notion in terms of denotation and signification, but if those semantic notions were also relative to the conceptual scheme then the desired objectivity would not be achieved. Now, what I contest in this argument is the assumption that the semantic relations of denotation and signification are in any interesting sense "relative to the conceptual scheme"; on my

[17] One might even be tempted to speculate that *all* scientific revolutions involve denotational or significational refinement, but I think that that is unlikely. Consider the sequence of revolutions leading from Thomson's theory of the atom to our own. In all these revolutions, it is hard to see any significational refinement in the term 'electron': there is nothing that Thomson can plausibly be said to have partially signified, beyond the set of electrons. Moreover, the kind of argument used against (HN) and (HΛ) rules out the possibility that Thomson's word signified "Thomsonian electrons" (things that satisfied all of Thomson's beliefs involving 'electron'), or that it signified nothing at all; the only alternative seems to be (iii), that the word kept the same denotation despite the radical change of theory. (Actually the matter becomes a good deal more complicated when one reflects that 'electron' might be referentially indeterminate today (cf. next paragraph); but even so, the above considerations do not completely lose their force in showing that scientific revolution is possible without denotational refinement.)

[18] "Reply to Davidson," p. 304, *Synthese*, XIX, 1/2 (December 1968): 303–305. This view is set out more fully in §6 of *Word and Object*.

view they are perfectly objective relations which hold between terms on the one hand and extralinguistic objects or sets of objects on the other. The existence of referential indeterminacy shows only that the relations of denotation and signification are not well-defined in certain situations, and that if we want to apply semantics to those situations we have to invoke the more general relations of partial denotation and partial signification. But these more general relations (like the less general ones, in the situations in which those less general relations are well-defined) are perfectly objective relations between words and extralinguistic objects (or sets of objects), and are not in any interesting sense "relative to the conceptual scheme."

This account of the differences between Quine's view of indeterminacy and my own view is meant only to be suggestive; I will provide a more detailed investigation of the differences, and an argument for my view as opposed to Quine's, in a sequel to this paper.[19] I also hope, in the sequel, to extend the above semantics to certain other sorts of indeterminacy which could not be discussed in this paper.

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NOTES AND NEWS

The Department of Philosophy of the Ohio State University regrets to announce the death of Arthur Gluck, on January 23, 1973. Mr. Gluck, a Columbus industrialist, endowed the Julius Gluck Memorial Library of Philosophy at Ohio State and contributed to the philosophy libraries of several other institutions. A constant supporter of philosophy since his graduation from Ohio State in 1920, Mr. Gluck was founder and first president of the Ohio State Philosophy Club and established the Bingham Award for excellent undergraduate work in philosophy at Ohio State.

The newly formed Society for the Advancement of American Philosophy plans two meetings during the Academic year 1973/74. The Society will hold a symposium entitled "Ethics" at the Eastern Division APA Meeting at Atlanta in December 1973. Then on March 1 and 2, 1974, the Society for the Advancement of American Philosophy will meet at Vanderbilt University. The topic for the two-day meeting will be "Human Rights and the American Tradition." Membership is open to anyone interested in American philosophy. Write to Joseph G. Grassi, Secretary, Department of Philosophy, Fairfield University, Fairfield, Conn., 06430.

[19] But sections III and V of my "Tarski's Theory of Truth" (*op. cit.*) are also of considerable relevance to our disagreement.