

# THE LAW OF EXCLUDED MIDDLE IS SYNTHETIC A PRIORI, IF VALID

by  
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## 1 Introduction

In the semantic anti-realist tradition deriving from the writings of Michael Dummett<sup>2</sup> there is a clear need to follow through the consequences of one's stance on sentence meanings as determinate and compositional, and as beholden to rules of logical usage.

The anti-realist presses broadly intuitionistic logical reforms; the realist resists these. The realist does not wish to be methodologically *deprived*. He is unwilling to give up the strictly classical inferences of classical logic. The anti-realist replies that the new level of methodological discipline is in fact liberating; and that the realist is not entitled to those strictly classical inferences anyway. The situation, however, has been clouded by a failure (on both sides) to appreciate exactly what is at stake. I intend to spell out here exactly how the realist can be allowed to have his cake, but also be asked to improve his manners when eating it. In doing so, however, I remain an anti-realist. I insist, with Dummett, on the manifestation requirement in the theory of meaning; and I advocate also even farther-reaching logical reform than is customary for a Dummettian.

The anti-realist's views on meaning invite one to rehabilitate the analytic/synthetic distinction. Moreover, his favoured theory of meaning is an inferentialist one. This affords the prospect of examining certain mathematical theories more closely to see whether some of their content is actually analytical in nature. Shouldn't we classify as analytic at least those theo-

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<sup>2</sup>On Dummett's characterization, semantic realism (for a discourse) is the view that every declarative sentence of the discourse is determinately true or false; and that this can be so, for a given sentence, independently of our means of coming to know what its truth value is. This is the brand of realism I shall be considering in this paper. The issue at hand is how best we should taxonomize its logical and epistemological ramifications.

rems whose proofs use only meaning-specifying rules<sup>3</sup> for the mathematical operators and predicates of the theory? In advancing an affirmative answer to this question, the anti-realist should be prepared also to abandon a dogma that has long held sway in discussions of analyticity: the dogma that no analytic principles could ever commit one logically to the existence of any entities. I would urge that this dogma is false; for it is an adequacy requirement on a theory of meaning that certain expressions be revealed as committing one, by their very meanings, to the existence of certain (necessary) existents.

The prospect is therefore opened up of (i) classifying various mathematical truths as analytic, even when they carry existential commitment (albeit only to *necessary* existents, such as natural numbers); and (ii) re-classifying various logical principles as synthetic if they cannot be justified, on an anti-realist account, as true by virtue of their meanings alone. We see, then, that there is an important continuing debate for realists and anti-realists to conduct over the consequences of their disagreement about the nature of logico-linguistic meaning.

In this paper I shall explore prospect (ii).<sup>4</sup> I shall argue that the Law of Excluded Middle is a synthetic principle.<sup>5</sup> This flies in the face of all the

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<sup>3</sup>Note that not all rules are necessarily meaning-specifying rules. The anti-realist's theory of harmony and separability places quite strong constraints on what can serve as a meaning-specifying rule. It is by no means the case that any old set of axioms 'implicitly defining' the notions involved in them count as meaning-specifying. (I include axioms as a special case of rules: they are rules with no premisses.)

<sup>4</sup>Prospect (i) has been explored in *The Taming of The True*, ch.8. (unpublished).

<sup>5</sup>This claim is hinted at in only one of Dummett's writings, so far as I have been able to determine; but it remains undeveloped, and its consequences unexplored. In 'Frege's Distinction between Sense and Reference', 1975, reprinted in *Truth and Other Enigmas*, at pp.121-2, he writes:

... [Frege's] first fundamental idea is that the condition for a complex sentence to be true depends solely upon its composition out of atomic sentences. ... *This* idea is not seriously challenged, at least by anyone accepting a Fregean syntax. ... the second fundamental principle of classical [Fregean] semantics [is] that the condition for the truth of each sentence is, determinately either fulfilled or unfulfilled. We can regard this as a metaphysical assumption — an assumption of the existence of an objective reality independent of our knowledge. We can, equally, regard it as an assumption in the theory of meaning, namely that we succeed in conferring on our sentences a sense which renders them determinately true or false.

Note that Dummett would himself appear to favour the view that bivalence is an assumption in the theory of meaning, rather than a more straightforwardly metaphysical claim.

conventional wisdom, among analyticity theorists, to the effect that all the so-called ‘logical truths’ of classical logic are analytic. For the anti-realist, logic is indeed analytic; but there’s much less to logic than meets the realist’s eye.

## 2 Analyticity

A sentence’s truth value is in general determined by its meaning and by how things are (‘the way the World is’):

$$\left. \begin{array}{l} \text{World} \\ \text{Meaning of } \phi \end{array} \right\} \longrightarrow \text{truth value of } \phi$$

The world cannot confer a truth value on a meaningless sentence; likewise, no meaningful sentence can have its meaning play no role at all in determining its truth value. But it is perfectly possible, in these general and abstract terms, for the truth value of a sentence to depend solely on its meaning, so that the World may, as it were, be ignored.<sup>6</sup> In such a case the sentence is analytic. If true, it will be so by virtue of its meaning alone, and will tell us nothing about the World. By contrast, if the sentence, though true, is not analytic — that is, if it is synthetic — then the World must have made a contribution to determining its truth. When the World has thus a look in, we have to look out: a true synthetic sentence is telling us something about the World.

This strikes me as the fairest way to generalize Kant’s own definition of analyticity, which all commentators agree is overly limited. Kant did not consider the many logically more complex kinds of sentences besides those of subject-predicate form. Yet at least some of these more complex sentences can also be analytic, in the spirit intended to be captured by Kant’s inadequate definition. It is not enough, for any completely general account of analyticity, to speak of “the predicate  $B$  [in a singular judgment of the form ‘ $A$  is  $B$ ’] [belonging] to the subject  $A$ , as something which is (covertly) contained in the concept  $A$ ”,<sup>7</sup> and to say that the judgment “[adds] nothing

<sup>6</sup>Robert Kraut has objected that no argument has been offered for this possibility. Here, then, is an argument of the kind required: the sentence “If Kraut is right, then Kraut is right” does not depend on the World for its truth value. Here’s another argument: “If Kraut is wrong, then Kraut is wrong” does not depend on the World for its truth value. Kraut should now see that there is, in fact, an infinity of arguments establishing the possibility claimed.

<sup>7</sup>*Critique of Pure Reason*, at B11.

through the predicate to the concept of the subject, but merely [breaks] it up into those constituent concepts that have all along been thought in it...".<sup>8</sup> The later, Fregean, definition of an analytic sentence as one which can be obtained from a logically true sentence by substituting synonyms, was motivated by the understandable need for greater generality.<sup>9</sup> Frege's more general definition tried to focus, quite rightly, on the matter of *meaning* (through the invocation of synonymy, or sameness of meaning); for that is what is at issue in Kant's talk of conceptual containment in the logically simple case of a singular judgment of subject-predicate form.<sup>10</sup> Yet one fair complaint that can be made about both the spirit and the letter of Frege's generalized notion of analyticity is that *it presupposes logic*, in the sense that it takes for granted some fixed logical system (in Frege's case, a classical higher-order logic) and appeals to its set of logically true sentences in *generating* a more inclusive set of analytic sentences. The logically true sentences themselves form a core within the wider set of analytic sentences.

Frege's method is more or less what Carnap followed in *Logical Syntax of Language*. The catch, though, was that Carnap still uncritically presupposed classical logic as the repository of unvarnished analyticity-by-virtue-of-logical-meanings.

Both Frege and Carnap, I would argue, had matters the wrong way round. We need, rather, a notion of truth-by-virtue-of-meaning (that is, analyticity) *from which* it will follow, non-trivially, that the logical truths of one's chosen logical system are indeed analytic. The proper order of business should therefore be: first, characterize the meanings of the logical operators; secondly, survey the synonymies; thirdly, work out what logic is justified by the meanings of the logical operators, and hence what sentences count as logically true; fourthly and finally, use the method of substituting synonyms to expand from the set of logical truths just determined (and now *legitimated* as analytic) to the set of analytic truths in general. The outcome of this method should be exactly the set of sentences for whose determination as true the World (as in the picture given above) drops out as irrelevant.

*Elucidatory remark:* Once we realize that (as Dummett has often stressed) the proper context for talk of meaning is talk of speakers' *knowledge of meaning*, the theory of meaning becomes an epistemology of linguistic understanding. So the notion of analyticity becomes epistemic, *but without*

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<sup>8</sup> *loc.cit.*

<sup>9</sup> *The Foundations of Arithmetic*, ed. & tr. by J.L.Austin, Oxford, 1959; at p.4.

<sup>10</sup>Notoriously, however, Frege's definition fails to confer analytic status upon 'All red things are coloured' and 'All bodies are extended'.

*collapsing to the notion of being knowable a priori.*<sup>11</sup> According to the epistemic conception, a sentence is analytic just in case one's grasp of its meaning alone should suffice, in principle, for one's being able to *constitute* a warrant for its assertion.<sup>12</sup> Note that I am not requiring that one who understands a given analytic sentence  $\psi$  should be able, by virtue of that understanding alone, to decide whether  $\psi$  is true, or false. Rather, I am requiring only that, if the understander is presented with a purported warrant or refutation of  $\psi$ , then she should be able to assess whether that construction is indeed what it purports to be — namely, a warrant (or a refutation) of  $\psi$ ; and that, having been presented with the right kind of warrant, the understander would be able to re-constitute it for herself by relying only on her grasp of meaning (hence, not on any of her empirical experience). For  $\psi$  to be analytic, the understander's assessment and re-constitution of a warrant for  $\psi$  should be able to proceed on the basis of her grasp of  $\psi$ 's meaning alone. In order for the understander of an analytic  $\psi$  to appraise purported warrants or refutations of  $\psi$ , and then to re-constitute a successful one establishing  $\psi$  as analytic, she should not require any sensory experience; nor any perceptual beliefs; nor have recourse to any theoretical beliefs that have themselves been formed, ultimately, on the basis of empirical evidence.

Note that these elucidatory remarks serve only to underscore the point that an analytic truth will also be *a priori*. As Kant himself said, “it would be absurd to found an analytic judgment on experience. . . . there is no need

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<sup>11</sup>It was Vittorio Hösle who made me mindful of this danger.

<sup>12</sup>The notion of epistemic analyticity, as it applies to logic, is not new. McDowell, for example, expresses the matter as follows, when he says that the verificationist conception:

requires a conception of the truths of logic, not as true solely in virtue of the senses of the logical constants — without assuming the principle of bivalence, there is no telling which sentences have that status — but as *knowable solely in virtue of the senses of the logical constants*. (‘Truth Conditions, Bivalence and Verificationism’, in G. Evans and J. McDowell, eds., *Truth and Meaning*, Oxford University Press, 1976, pp.42-66; at p.60. My emphasis.)

The term ‘epistemically analytic’ comes from Boghossian’s paper ‘Analyticity’, forthcoming in Crispin Wright and Bob Hale (eds.), *A Companion to the Philosophy of Language*, Oxford, Blackwell, 1995. McDowell’s formulation of epistemic analyticity, though, masks an important ambiguity, which is neither noted nor appreciated by Boghossian, *loc.cit.* We need to distinguish between grasp of meaning which would ensure the discovery of a warrant to assert, and grasp of meaning which would suffice merely for the constitution (or re-constitution) of a warrant to assert. The latter (weaker) notion is to be preferred. That is, a sentence  $\phi$  is epistemically analytic just in case (a) there is a warrant  $\Pi$  for its assertion, and (b) grasp of  $\phi$ 's meaning suffices, in principle, for one to re-constitute  $\Pi$  as a warrant for  $\phi$ 's assertion, were one ever to be made aware of  $\Pi$ .

to appeal to the testimony of experience in its support.”<sup>13</sup> Kant, in making this remark, was certainly not identifying analyticity with aprioricity.

Nor do we intend to conflate analyticity with aprioricity. Analyticity remains for us, as for Kant, a matter of meaning. *A priori* knowledge, by contrast, remains for us, as for Kant, knowledge that is *absolutely independent of all empirical experience*. This leaves open the possibility of synthetic *a priori* judgments. With our understanding of the *a priori* status of any judgment, however, we eschew appeal to Kant’s problematic criteria of absolute necessity and strict universality. We work instead only with the central negative definition that emphasizes independence from empirical experience.

Now Kant’s favourite and *prima facie* most secure kind of synthetic *a priori* knowledge is mathematical knowledge, based on formal proofs employing abstract constructions. At B15-16 in the *Critique of Pure Reason* he passed from the too swiftly established syntheticity of the simple arithmetical statement “ $7+5=12$ ” to the sweeping conclusion that arithmetical propositions are always synthetic. There is a strong case, however, for saying that some areas of mathematics — such as number theory — contain epistemically *analytic* statements. Indeed, once one has properly analyzed how arithmetical knowledge is generated by appropriately chosen (and meaning-constituting) *rules*, one is in a strong position to gainsay Kant even on his chosen example “ $7+5=12$ ”. The rules in question are ones that Kant himself never had in mind, and was never able to consider. They figure in what I have elsewhere called a ‘constructive logicist’ derivation of the Peano-Dedekind axioms, in the spirit of Frege’s foundational approach to arithmetic.<sup>14</sup> These rules are perfectly meaning-constituting and meaning-respecting as far as the arithmetical terms are concerned. Therefore, since they suffice for a proof of “ $7+5=12$ ”, that sentence is analytic.

Not only can some sentences of arithmetic be analytic; they can also carry existential commitments. These commitments are not vicious, though, since the (abstract) entities involved — namely, the natural numbers — are necessary existents. Epistemically analytic statements need not be any of the following: decidable; obvious; free of existential commitments.<sup>15</sup>

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<sup>13</sup> *CPR*, B11-12.

<sup>14</sup> See *Anti-Realism and Logic*, ch.XX.

<sup>15</sup> Anyone who maintained that first-order logical truths are analytic is already committed to the view that analytic statements need not be decidable or obvious. But it is decidable and obvious that analytic statements need not be free of existential commitments either: consider the analytic claim that the number of non-self-identical things is 0. In *The Taming of The True*, ch.8, an account of analyticity is developed that warrants

This is not to say that *all* of mathematics turns out (*pace* Kant!) as analytic rather than synthetic. It is only to say that at least *some* mathematical propositions may well be analytic rather than synthetic, while yet others remain synthetic. Only a closer, careful look will tell us exactly which ones are which.

*Mutatis mutandis*: perhaps not all of the so-called ‘logical truths’ (of classical logic) are analytic. Let us now examine this possibility more closely.

### 3 The previous status of the Law of Excluded Middle

On standard accounts of the analytic/synthetic distinction, all the so-called ‘logical truths’ of classical logic are held to be analytic. That is why Frege’s logicist reduction of classical arithmetic — had it succeeded — would have shown arithmetic to be analytic after all, and not, as Kant thought, synthetic. Anyone who accepted the analytic/synthetic distinction as tenable and useful would have been inclined to give as examples of analytic principles of logic both the Principle of Non-Contradiction ( $\neg(\phi \wedge \neg\phi)$ ) and the Law of Excluded Middle ( $\phi \vee \neg\phi$ ). This co-classification is one that would have survived every one of the major revisions proposed for the contents of ‘Kant’s Boxes’:

	<i>a priori</i>	<i>a posteriori</i>	
analytic	Logic	X	
synthetic	Arithmetic Geometry		KANT’S BOXES

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all of these claims.

As is well-known, the Logical Positivists sought first to evacuate the synthetic *a priori* box. They were confident in the ‘logician’ reduction of arithmetic afforded by Russell and Whitehead’s *Principia Mathematica* (so that arithmetic was re-classified as analytic); and they were impressed by the role played by non-Euclidean geometries in relativistic physics (so that geometry was re-classified as *a posteriori*). Next came Gödel’s first incompleteness theorem for arithmetic, causing those who still held to the analytic-synthetic distinction to re-consider the Logical Positivists’ classification of arithmetic as analytic. Perhaps arithmetic was synthetic after all? But then came Quine’s ‘Two Dogmas of Empiricism’, and it was no longer fashionable to think about arithmetic (or any other area of discourse, for that matter) in these terms.<sup>16</sup>

## 4 The Law of Excluded Middle and the Principle of Bivalence

The reforming anti-realist insists on manifestationism, and holds that logic should be analytic. His analysis of meaning renders only broadly intuitionistic logical principles as analytically acceptable. Accordingly, he rejects or eschews as *logical* principles all strictly classical principles of inference that do not belong to the justified canon. He is committed to the view that the latter principles, if necessarily true or necessarily truth-preserving, are *synthetic*. That, indeed, is the thesis to be defended here: strictly classical logical principles are synthetic *a priori*, if valid). Note that this is, in a certain sense, a *vicarious* claim put forward by the perspicuous anti-realist, ‘on behalf of’ the realist. The anti-realist is not going to accept the strictly classical logical principles; so, in particular, he is not going to claim, *himself*, that they are *a priori*. Rather, he is offering this Kantian perspective to the realist in order for the latter to make better sense of *her* own commitment to those principles.<sup>17</sup> That is why my title is conditional: the Law of Excluded Middle is synthetic *a priori*, if valid.<sup>18</sup>

<sup>16</sup>For a detailed discussion of the vicissitudes of classification, and of the tenability of the analytic-synthetic distinction itself, see *The Taming of The True*, ch.8. In this brief sketch I am perforce ignoring the subtlety of distinguishing between formal and physical geometry; and I am ignoring also the fate of other Kantian synthetic *a priori* principles besides those of arithmetic and geometry, such as the Principle of Causation.

<sup>17</sup>I am grateful to Alan Code for eliciting this clarification.

<sup>18</sup>Stewart Shapiro has asked (private communication) ‘on what grounds can the realist claim to know *LEM* a priori? The fact that he *believes* the world to be determinate does

To establish the thesis I shall, as the title suggests, concentrate on the classicist’s Law of Excluded Middle:

$$(LEM) \quad \overline{\phi \vee \neg\phi}$$

according to which every instance of the displayed schema is a necessarily true sentence of the object language, whose (necessary) truth is known *a priori*. I shall proceed on the assumption that this schematic law gives exact expression, within the logic, to the metalinguistic Principle of Bivalence:

$$(Biv) \quad \text{For every sentence } \phi \text{ of the object language, exactly one of } \phi, \neg\phi \text{ is true.}$$

I am aware that there are supervaluational systems, say, for which Bivalence does not hold but in which the Law of Excluded Middle is nevertheless held to be correct. Such systems shall be set aside as irrelevant for the argumentative concerns of this paper. If need be, I shall address my argument only to those classicists for whom *LEM* gives precise expression, within their system, to *Biv*.<sup>19</sup>

Even so, one can still acknowledge an obvious difference between the Principle of Bivalence and the Law of Excluded Middle. It is this:

- *Biv* is a single statement, in the metalanguage; it is an explicit universal generalization about all sentences of the object language.
- *LEM* is a schema, with infinitely many instances in the object language. Each instance is a logical axiom for the classical logician. So *LEM* is really an infinite set of (classical) laws in the object language, all having the same form  $\phi \vee \neg\phi$ . In speaking of the Law of Excluded Middle, one is saying that any such instance  $P \vee \neg P$  is held to be true *a priori*, prescindingly from what we may know as to the truth value of *P* itself.

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not give him license to claim that *LEM* can be *known* independently of experience. But we need knowledge or knowability to get a priority.’ That, I would want to say, is the realist’s problem, not mine. The realist may in the end have to settle for saying something like ‘Look, the spade simply turns with *LEM*, and I know it.’ The imaginative work being done here on behalf of the realist (or, at least, in the interests of a better understanding of what realism amounts to) is aimed at securing classical principle such as *LEM* a ‘most favoured possible status’. If an objector wished to suggest that *LEM* be regarded, rather, as *a posteriori* rather than *a priori*, well then — well and good! The cards would at least then be out on the table; and the strictly classical parts of logic would no longer be among the face cards.

<sup>19</sup>I owe to Stewart Shapiro the observation that *LEM* is equivalent to *Biv* if the Tarskian T-sentences are assumed. So the supervaluationist has to demur from the latter.

We are maintaining that these instances  $P \vee \neg P$  are better thought of as being held to be (necessarily) true on *a priori* rather than on ‘logical’ grounds. Indeed, the holding true (as a matter of necessity) of every such instance (or the holding true, as a matter of necessity, of any such instance  $P \vee \neg P$ , absent any proof or refutation of  $P$ ) expresses an essentially *metaphysical* belief. This belief is to the effect that *the world is determinate* in every expressible regard (or at least, in the case of a particular instance of *LEM*, determinate in the respects answering to the propositional content  $P$  involved). Such a belief is *synthetic*, since its content cannot be known to be true simply on the basis of the meanings of the logical expressions  $\vee$  and  $\neg$ .<sup>20</sup>

## 5 Logic is analytic; but which logic is the correct logic?

Logic is a canon of inference, consisting of laws and/or rules of inference, whose justification turns on questions of *meaning* alone. That is, which logic is the correct logic is an *analytic* matter. The correct logic, it turns out, is *intuitionistic relevant logic (IR)*.<sup>21</sup> Its correctness derives from the fact that its rules of inference are exactly faithful to the meanings of the usual logical operators ( $\neg$ ,  $\wedge$ ,  $\vee$ ,  $\supset$ ,  $\exists$ ,  $\forall$ ). The meanings in question are those that can be grasped on the basis of observable behaviour, and whose grasp can be manifested fully in observable behaviour. In short, it is by virtue of a theoretically demanding *analysis of meaning* that intuitionistic relevant logic is revealed as the correct logic. That is, the laws of intuitionistic relevant logic (or the inferences permitted therein) are *analytic*; and, indeed, these laws and inferences are all the analytic laws and inferences there are.

(*IR*) results from two lines of reform against the full classical canon. The first one *intuitionizes* or *constructivizes* the logic by paying heed to epistemic constraints on truth. Truth is knowable, and consists in the existence of an

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<sup>20</sup>Stewart Shapiro has suggested (private communication) that ‘LEM is largely epistemic. It does not (only) say that the world is determinate, but that it is cooperative. Every proposition is either knowable or refutable.’ My response here would be: yes, but what do you think it is about the world that is supposed to make the latter epistemic claim true? Wouldn’t it be precisely the alleged determinacy of the world? (One has to bear in mind here that ‘knowable’ means ‘knowable in principle’, which involves a certain amount of idealization.)

<sup>21</sup>An impression of impudence in making this claim might be dispelled by *Anti-Realism and Logic*, Clarendon Press, Oxford, 1987; and *The Taming of The True*, forthcoming.

effectively checkable construction, or warrant, establishing the sentence in question as true. This leads one in the direction of an intuitionistic logic as the logic of such constructions.

The second line of reform *relevantizes* the logic by paying heed to an epistemic constraint on logical consequence or deducibility. The epistemic constraint in question can, very briefly, be explained as follows. Suppose that as we search for a proof (call it  $\Sigma$ , say) of a given argument  $\Gamma : \psi$ , we have made some progress and have reached a stage at which we have posed for ourselves the deductive sub-problem  $\Delta : \phi$ . We are now looking for a proof  $\Pi$ , say, for this sub-problem.  $\Pi$  will then feature as a sub-proof of  $\Sigma$ . Suppose that we are fortunate enough, subsequently, to find a proof  $\Pi'$  of a *stronger* result  $\Delta' : \phi$  or  $\Delta' : \emptyset$  (where  $\Delta'$  is a *proper* subset of  $\Delta$ ). The Principle of Non-Forfeiture of Epistemic Gain is to the effect that we should be able to use the proof  $\Pi'$  of the stronger result (*instead of* the proof  $\Pi$  of the weaker result) as a sub-proof within  $\Sigma$ , without thereby destroying the proofhood of the overall proof  $\Sigma$  itself. Moreover, the overall result obtained by placing  $\Pi'$  within the wider proof environment  $\Sigma$  already built up (and on the assumption, of course, that all other deductive sub-problems are solved positively) should be a proof of  $\Gamma' : \psi$  or  $\Gamma' : \emptyset$ , for some subset  $\Gamma'$  of  $\Gamma$ . That is, the overall result established by  $\Sigma$  (when it contains  $\Pi'$  as a subproof) should be at least as strong (logically) as the would-be result  $\Gamma : \psi$  for which a proof was being sought in the first place. The epistemic constraint on deducibility is that *logic's labour should not be lost*. This principle of non-forfeiture uniquely determines *IR* as the correct fragment of intuitionistic logic.<sup>22</sup>

## 6 On ideal justifications

The reforming anti-realist is not so much intent on depriving the realist of his classical tools; but, rather, on having the realist acknowledge the true nature of his use of them, or of his appeal to them. It is a methodological ideal, going back to Aristotle, that the provenance of any knowledge claim should admit of regimentation. That is, the justificatory pedigree of any knowledge claim should be able to be laid bare in all its details, open to critical inspection and appraisal. Strictly logical steps of inference should be able to be labelled as such; inductive leaps likewise. Sometimes the local appropriateness of a particular step might be legitimated only by appeal

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<sup>22</sup>For more details, see *The Taming of The True*, ch.9.

to more global considerations about sensitivity to the available evidence, and the use of explanatory foils (as in the case of inference to the best explanation<sup>23</sup>). But, so the venerable ideal goes, we should be able to get absolutely clear just what we are up to at each and every inferential step on the way to a new knowledge claim.<sup>24</sup>

In this context, what I am recommending here is this: the realist should be prepared to identify every so-called ‘logical’ step in any passage of his reasoning that is not, on our strict anti-realist view, analytic; and must be prepared to label such steps as *synthetic*. ‘Logical’ steps that are not really analytic are (by definition) synthetic; hence, should be acknowledged as such. If (and only if) this is done, we can always be clear as to how and when the justification of a knowledge claim involves recourse to an essentially *metaphysical* conception of reality. The importation of a metaphysical conception of reality (such as that of the realist) is something that goes strictly beyond the adducing of observable evidence, or the plying of meaning-respecting routes of logical (or mathematical<sup>25</sup>) reasoning. When the realist uses the Law of Excluded Middle on some instance  $P \vee \neg P$ , he must be prepared to acknowledge: “Here I presuppose, and give expression to, the metaphysical view that reality is determinate (in the respect  $P$ ).”<sup>26</sup>

The benefits of regimentation are not to be underestimated. Many ‘simple’ steps in ordinary classical reasoning, such as, say:

$$\frac{\neg \forall x F(x)}{\exists x \neg F(x)} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

are compounds of the simpler steps that count as basic in a system of natural

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<sup>23</sup>See the illuminating discussion in Peter Lipton’s *Inference to the Best Explanation*, Routledge, 1991.

<sup>24</sup>This requirement of ‘perfectibility of proof’ applies as much to a coherentist epistemology as it does to a foundational one. For even the coherentist is appealing to the logical inter-relatedness of sentences in her system when showing that all her beliefs *cohere*. And if she needs classical logic to do this, then she is basing coherence itself, at least in part, on the assumed determinacy of the world; so even she should think again about the metaphysical implications of her methodological norms.

<sup>25</sup>Remember that it is a consequence of our re-fashioning of the analytic-synthetic distinction that a great deal of mathematical inference is revealed as analytic. Indeed, we even allow that an analytic truth might entail the existence of certain entities, albeit only *necessarily existing* entities. For more details, see *The Taming of The True*, ch.8.

<sup>26</sup>Along with the Law of Excluded Middle, there are other, equivalent, classical negation rules whose application would call for a similar acknowledgement. They are the Law of Double Negation, the rule of Classical Reductio, and the rule of Dilemma.

deduction. Among the latter are the rules of introduction and elimination of the logical operators, *plus* — in the classical case — at least one strictly classical rule of negation. In everyday reasoning, which goes unregimented (or insufficiently regimented), the full brunt of the ‘classicism’ borne by the latter negation rules is obscured. It will not strike the logically untutored eye that it is just one isolable kind of strictly classical step that is persistently responsible for the classical nature of the reasoning throughout. Given that negations co-occur with the other logical operators in complex patterns among which we navigate by the cruder likes of the compound moves just displayed above, it is all too easy and inviting to view all the logical operators as imbued or tinged with some imagined ‘classical ingredient’ of meaning. Given that that is indeed so, one can say that the finer analysis of the structure of logical deduction afforded by the work of Gentzen has not yet been milked for its final philosophical insights. Among these, I contend, is how the contribution of a realist metaphysical outlook can be logically *corralled*. It can be separated off from the other basic rules, all of which contribute to the exhaustive specification of the meanings of the logical operators.

The realist should not be embarrassed by the recommendation above concerning regimentation. For there should be nothing to hide, and indeed much to be gained, from being forthright and clear about exactly where one’s metaphysical conception intrudes into proffered justifications of knowledge claims. The realist does not shrink from asserting his realism; so why, then, should he be at all reluctant to concede that it would be quite in order to *track the epistemic impact* of his realism across the justifications of his knowledge claims? Nor, if the realist is willing to do this, should the anti-realist wish, any longer, to deprive him of anything. For the anti-realist would now be confronted with a range of knowledge claims and their respective justifications, in which any realist metaphysical tinctures have openly left their stain. It would be up to the anti-realist to treat these justifications with the requisite caution. Their conclusions could be ‘bracketed’, or seen as depending — at least insofar as these stained justifications of them are concerned — upon the metaphysical conception of the realist.

## 7 Epistemic analyticity and inextricability

Now of course the Quinean will complain at this point that we are underestimating the problem of (alleged) inextricability of belief and meaning. The

Quinean will demand watertight behavioural criteria by means of which one will be able to ‘factor out’ the respective contributions of (factual) belief and of (conceptual) meaning to the determination of the truth value of any sentence. And the Quinean will complain that one cannot, in principle, effect such a factorization: not, at least, if one accepts that all there is to the grasp of meaning is what can be disclosed in observable behaviour.

The anti-realist’s response to the Quinean is as follows. The behavioural manifestation requirement is precisely what leads the anti-realist to his account of the meanings of the logical operators. That account is informed by the further requirement of *harmony* between the conditions justifying an assertion, and the commitments on which an audience can rely, when the assertion is made.<sup>27</sup> This harmony requirement finds natural expression, in the case of any logical operator, in the reduction procedure associated with the operator. (It is the repeated application of these reduction procedures that enable one to turn any given proof into one in normal form.<sup>28</sup>) The logic determined by the rules displaying such harmony is what turns out to be analytically justifiable. Moreover, we can reasonably require, for attribution of grasp of meaning, that the speaker be able to manifest that grasp fully in observable behaviour *by the exercise of suitable recognitional capacities*. Careful examination of the various ways in which the italicized phrase can be made more precise reveals that one cannot hold both to Bivalence and to Manifestationism — at least for any *undecidable* discourse.<sup>29</sup> Hence, for any undecidable discourse (such as that of arithmetic) the Quinean, by virtue of being a manifestationist, should eschew Bivalence!

As for the objection that the factual and conceptual contributions to the determination of the truth value of a sentence are inextricable, my reply is that any ‘extrication’ on offer would be a defeasible, interpretative hypothesis. There is a valuable role for such interpretative hypotheses. Like any other high-level scientific hypothesis, they perch gingerly atop a mountain of supporting, but logically inconclusive, behavioural evidence; and they are liable to be toppled as new evidence accrues. But the distinction they

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<sup>27</sup>Although Brandom has picked up on the harmony requirement in his book *Making It Explicit*, Harvard University Press, 1994, he does not follow through, as Dummett does, the implications this may have for logical reform. Brandom remains a quietist about classical logic, for want of a more detailed examination of what the requirement of harmony, properly understood and prosecuted, entails.

<sup>28</sup>For a fuller account of harmony, reduction and constructive validity of arguments, see *Anti-Realism and Logic*.

<sup>29</sup>For a detailed argument for this claim, see *The Taming of The True*, ch.6.

postulate — namely, a distinction between a *semantic* contribution from within the language, and a *factual* contribution from non-linguistic, worldly states of affairs in the determination of a sentence’s truth value — is one that we would wish, in principle, to be able to make. Part of the test of the overall acceptability of an interpretative theory is just how the sentential chips would fall with respect to the distinction as finally settled. *That there is* such a distinction to be made is presupposed by the very project of interpreting linguistic utterances. That there might be disagreements as to where, precisely, such a distinction is finally to be drawn is to be expected from the nature of the enterprise. That interpreters in disagreement would wish to draw the distinction in different places should not surprise one, since the precise placement of the distinction informs the particular interpretation that any one interpreter favours. If one is in the business of imputing contents (to linguistic utterances and/or mental states), then one is also in the business of trying to account for when sentences are asserted (or mental states occupied) wholly by virtue of the speaker’s grasp of the contents with which they are laden, and not by virtue also of some worldly input or opinion. That is to say, the question of where to draw the analytic/synthetic distinction is firmly on the agenda, even within the project of radical interpretation.

Note also that the Quinean confines himself to the role of the *alien observer*: one encountering the linguistic community from the outside, having had no experience within it. If, however, one adopts the standpoint of the *reflective participant* in a linguistic practice, one will have sharper intuitions about the precise demarcation of the analytic/synthetic divide.

## 8 How to inflate the correct logic to obtain classical logic

Intuitionistic relevant logic (*IR*) is properly contained within both intuitionistic logic and classical logic. Classical relevant logic (*CR*) is obtained from *IR* by adding the Law of Excluded Middle. Full classical logic (*C*) is then obtained from classical relevant logic by adding the rule of *ex falso quodlibet*, according to which a contradiction (or absurdity) implies any sentence whatsoever:

$$(EFQ) \frac{\perp}{\phi}$$

Equivalently, one could formulate *EFQ* as a single metalinguistic principle, to the effect that any contradiction entails any proposition whatsoever.

*IR* is the canon of inference that is based fully on the meanings of the logical operators; and those meanings are exhausted in sustaining just the inferences afforded by *IR*. The system *CR* of classical relevant logic can be thought of as a canon of inference that is based partly on the meanings of the logical operators, but residually — and very importantly — on an extremely high-level synthetic principle: the metaphysical principle that reality is determinate. Moreover, the principle is claimed to be known *a priori*, and would indeed be *a priori*, if it were valid. In understanding and classifying matters this way, the anti-realist of course cannot *endorse* this principle; indeed, he eschews it. So the principle cannot be *a priori* for the anti-realist. Rather, the anti-realist has to represent the principle as *a priori for the realist*. He does so by way of best explanation of the realist's persistent use of the principle in his classical reasoning. For the realist certainly does not treat the principle as, say, an inductive generalization based on past experience. Rather, the realist asserts the principle with no justification at all. This is meant here in two dovetailing senses. First, the realist has no justification (and reprehensibly so), from the anti-realist's point of view, for his reliance on the principle; and secondly, the realist, for his part, denies that any justification *need* or *could* be provided. Ironically, what Kant said about natural laws at B198 could be adapted here: "Even [the principle of determinacy], viewed as [a principle] of the empirical employment of understanding, [carries] with [it] an expression of necessity, and so [contains] at least the suggestion of a determination from grounds which are valid *a priori* and antecedently to all experience." With the principle of determinacy, however, one might urge that there is more than just a *suggestion* to this effect, so far as the realist is concerned; but that the envisaged determination is degenerate or trivial.

Even for the Kantian, *LEM* would not need any transcendental deduction of its *a priori* concepts, for there *aren't any a priori* concepts specific to *LEM*! The logical concepts of disjunction and negation are governed by their meaning-specifying introduction and elimination rules, and that is all that needs to be said on that score. Naturally, we are entitled to use those logical operators, and there's the end of it, as far as any transcendental deduction of *concepts* goes.

Interestingly, however, the same cannot be said for the Principle of Bivalence. The latter principle *makes explicit*, at the meta-level, what is only implicit in the classicist's use of *LEM* (via its various instances) in his de-

ductive reasoning. The move to explicitness comes with the introduction of the concept of truth: Bivalence tells us that every declarative sentence is either true or false (equivalently: either the sentence or its negation is true). What, then, about the need for a transcendental deduction of the *a priori* concept of truth itself? Fortunately, that would be a problem for the realist who wishes to make matters explicit, not for the anti-realist. But even the realist has the option of refusing to be drawn into making matters explicit. He could, if he wished, stay resolutely within the language governed by *LEM*. He need not invoke the notion of truth in any attempt explicitly to render, in a single statement, what his allegiance to *LEM* (through his multiple applications of it) amounted to. It is the realist who cleaves to *LEM* without codifying that commitment explicitly in the form of Bivalence who lends himself most clearly to the construal being proposed here: namely, that his use of *LEM* is expressive of his metaphysical outlook, and that within his system *LEM* should be regarded as having the status of a synthetic *a priori* principle.

## 9 The status of EFQ

According to the realist's principle of determinacy, the truth value of any declarative statement is determined by reality in advance of our investigations; and that truth value could attach to the statement quite independently of us and our beliefs, and independently also of our available means of coming to know what is the case. To the extent that this account of metaphysical realism seems merely metaphorical, it is Dummett's suggestion that one take the espousal of the Principle of Bivalence to be the more precise and definitive expression of the metaphysical view in question. Since we are here equating allegiance to Bivalence with adoption, in one's logic, of the Law of Excluded Middle, we shall equally regard *a priori* reliance on the latter law as fully expressive of the realism in question. The system *CR* is really *all* that the classicist needs for any of his deductive purposes. He does not need the full system *C* of classical logic. For we have a metatheorem to the effect that

$$\Delta \vdash_C \phi \Rightarrow \text{for some subset } \Delta' \text{ of } \Delta, \text{ either } \Delta' \vdash_{CR} \phi \text{ or } \Delta' \vdash_{CR} \perp$$

This metatheorem tells us that, given any classical result, *CR* can produce that result or possibly an even stronger one (namely, by not using all of the

given premisses, or indeed showing that some of the premisses are jointly inconsistent). Thus there is good reason to complain that the extra irrelevantist trappings of the full system  $C$  serve only to obscure the essential logical connections that even the classicist would want to bring out between the premisses and conclusion of a classical argument. How are these irrelevantist trappings acquired?

The system  $C$  of full classical logic goes even further than  $CR$  by adopting *ex falso quodlibet*. This inferential principle is the precise expression of an epistemological view, for which I shall now try to articulate a more metaphorical understanding — parallel to the way that the precise  $LEM$  could be rendered, metaphorically, as the metaphysical picture just given above.  $EFQ$ , figuratively speaking, says that our thinking about reality is a seamless whole — that any one bad apple will ruin the whole barrel. It warns one that an absurdity encountered or generated in one's thinking in any one area of cognitive activity immediately spills over to contaminate one's thinking in *any other* area.  $EFQ$  says that there are no insulable contradictions. It says that incoherencies cannot be quarantined off.  $EFQ$  speaks of a true disaster-mentality. It says that every little spill of contaminant will poison the water table of the whole planet, and indeed that of every other planet in the universe. It says that any local muddle in one's thinking entails the global collapse or trivialization of all one's theories concerning all other subject matters.  $EFQ$  says that there is really only one inconsistent theory — namely, the whole language.

The opponent of  $EFQ$ , however, can point out that it is bad enough to encounter a contradiction, locally, in one's system of beliefs. Discovering that absurdity lurks in one's thoughts about some subject matter is in itself sufficient reason to diagnose the problem — *locally* — and apply an appropriate remedy to one's thought about the subject matter in question. It is quite unnecessary to be prompted to undertake such revision by having some epistemic sword of Damocles, in the form of  $EFQ$ , dangling above one's already chastened and willing head.

It is rather ironic that the realist who is given to speaking of carving reality at the joints does not seem to appreciate that with joints there come relatively disjoint domains of things, properties and facts: domains among which  $EFQ$  is not just an inferential licence, but is inferentially licentious.  $EFQ$  strikes one as carrying more of an *epistemological* import than a *metaphysical* import. It says something very general about one's *thinking* or *theorizing*, rather than about reality itself. This is for the simple reason that reality itself (*pace* Hegel and the latter-day Priest) cannot harbour any

contradictions. So *EFQ* cannot even be interpreted as a universal law about ideal types of states of affairs.<sup>30</sup> If we are looking for a picture, then, to capture the import of *EFQ* as a logical principle, it would have to be something more like *True thoughts about the World form a seamless web* rather than *Reality itself is a seamless whole*.

Determinacy *in rebus* and seamlessness *in cogitationibus* are the two pictures rejected by the anti-realist who regards *IR* as the correct logic on analytic grounds. The metaphysical and epistemological picture that arises from eschewing both *LEM* and *EFQ* is that of states of affairs being determinate only to the extent that they are, in principle, knowable; and of theories being sufficiently discomfitted by the production of a contradiction to warrant immediate and *local* revision. One can acknowledge the urgent need for such revision without subscribing to the dogma, absurd (though not self-contradictory!) in itself, that, if left untouched, such a contradiction in one's theory would entail epistemic commitment to any and every statement within that local discourse and within every other discourse, no matter how disconnected or disparate its subject matter may be.

*EFQ* is not analytic. Any truth which is not analytic is synthetic. True synthetic principles tell us something about the World. Thus *EFQ*, if true, tells us something about the World. But we have just satisfied ourselves that *EFQ* is really an *epistemological* principle, telling us something about the structure of supposedly rational *thought about* the World. The conclusion invited is therefore that the proponent of *EFQ* is committed to including the structure of rational thought as itself part of the World. *This* would be once again to see it as a metaphysical principle.

But the *opponent* of *EFQ* says that, far from being true, *EFQ* is false — indeed, he would like to say, *necessarily* false. Isn't it just patently obvious that it is not the case, on a pre-formal and intuitive understanding of entailment, that any contradiction entails every proposition whatsoever? This should be an adequacy constraint on any formal explication of the notion of entailment, and on any formulation of a system of deduction intended to do justice to that pre-formal and intuitive notion.<sup>31</sup> Accordingly, one can

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<sup>30</sup>It is not logically possible for a world to contain a contradiction. But ideal types of states of affairs — ones containing, say, perfectly spherical solids, frictionless and perfectly plane surfaces, perfectly straight and rigid rods etc. — are logically even if not physically possible.

<sup>31</sup>Thus, one may be totally unmoved by objections to the effect that such-and-such well-known modal or model-theoretic definition of logical consequence has *EFQ* as a necessary and 'idealizing' consequence concerning logical consequence itself. *EFQ* will always be

maintain that there are indefinitely many straightforward counterexamples to *EFQ*. An inconsistency in one's thought about Julius Caesar — say, implying that he had sixteen teeth on his death, and implying also that he had seventeen teeth on his death — cannot entail commitment to the view that the moon is made of Camembert, or that the last television commercial beamed out before the Earth explodes will be from Coca-Cola. This makes *EFQ*, in addition to being a red herring, a very different kettle of fish from *LEM*. For *LEM*, while not necessarily true, is immune to counterexample. That is why the anti-realist has to be careful simply to avoid commitment to *LEM*, rather than denying it.

## 10 Some ironies, and a new understanding

I would like to point out some ironies that have emerged from this discussion.

First, the very behaviourism in the theory of meaning that is urged by Quine is expressed by the Dummettian anti-realist in the form of the manifestation requirement. This requirement leads one to embrace the analytic/synthetic distinction, rather than reject it.

Secondly, one of the favourite, paradigmatic examples, for a Kantian, of synthetic *a priori* principles — *LEM* — turns out to be synthetic.

Thirdly, many of the paradigmatic examples, for a Kantian, of synthetic *a priori* principles — such as “ $7+5=12$ ” — turn out to be analytic. Mathematical intuitionism, given its Kantian roots, could well be expected to be synthetic — but, for the Dummettian, intuitionistic logic at least, and arguably also much of mathematics (such as Heyting arithmetic, that is, the intuitionistic Peano-Dedekind theory of the natural numbers) turns out to be analytic. It is the strictly classical residue of classical logic which, for the Dummettian, turns out to be synthetic!

The final irony is this: if the argument given here succeeds, we shall have extricated, in the case of *LEM*, the (anti-realistically licit) strand of *meaning* from the strand of (metaphysically realist) *belief* motivating its acceptance. And we shall have done so in the case of a principle which, for the Quinean with his quietist acceptance of classical logic, would have been analytic if any principle had been capable of being so.

found to be at work at the meta-level in these ‘justifications’ of *EFQ* at the object-level.

## 11 Re-construing the dispute between the classicist and the intuitionist

To the extent that Kant wanted to find a proper role for metaphysics by insisting on the existence of synthetic *a priori* principles, he missed a great opportunity by not singling out *LEM* as prime among these. (But of course in his day a proper understanding and systematic formulation of classical logic, especially as a system of natural rules of inference, was not yet available.) *LEM* is much more interesting than any particular mathematical truth such as “ $7+5=12$ ”. It has a high-level and very general import, and on reflection strikes one as one of the most ‘metaphysical’ principles one could countenance. It does not require, for the explanation of its *a priori* status, any of the further controversial doctrines about the respective roles of the sensibility and of the understanding; nor does it involve maintaining any dogma about a scheme/content distinction.

The dispute between the realist and the anti-realist, we have suggested, can be alleviated by having the realist acknowledge, when using *LEM*, that it expresses an *a priori* metaphysical conviction on his part. It is not analytic; and of course it cannot be thought to be held *a posteriori*, given the nature of its use in classical reasoning. So it is synthetic *a priori*, if universally true. And, strangely enough, though he swear no allegiance to it, the anti-realist cannot, for his part, claim that any instance of *LEM* is false.<sup>32</sup>

Finally, regarding *LEM* as synthetic *a priori* provokes an interesting and far-reaching re-consideration of the exact nature of the dispute between the realist and the anti-realist (or between the classical mathematician and the intuitionistic mathematician) over the correct choice of logic. Dummett would have it that the classicist wants to regard the intuitionist as treating the logical operators as having unusual, ‘constructive’ meanings, which are distinct from their more usual ‘classical’ meanings. Moreover, according to Dummett, the intuitionist would object that *there just aren’t any classical meanings* of the kind that the classicist imagines he is able to attach to the logical operators.<sup>33</sup> According to Dummett, the classicist charges the

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<sup>32</sup>For a more careful canvassing of the possibilities open to the anti-realist with respect to Bivalence, however, see *The Taming of The True*, ch.6.

<sup>33</sup>In *The Logical Basis of Metaphysics*, Harvard University Press, 1991, Dummett writes (at p.17):

The affront to which the [intuitionist’s] challenge [to fundamental accustomed modes of reasoning] gives rise is quickly allayed by a resolve to take no notice. The challenger *must* mean something different by the logical con-

intuitionist with change of subject matter; while the intuitionist charges the classicist with consorting with incoherent, non-existent meanings.

We are now in a position to see that both of the charges that Dummett represents the two sides as laying would be wrong. On the view defended here, there is no ‘change of subject matter’ (concerning ‘logical meanings’ of the operators); nor should the classicist be denied the meanings actually borne by the logical operators even as he presses them into strictly classical patterns of inference. For we have seen that there really are only the meanings answering to the anti-realist’s constraints, meanings that legitimate exactly the system *IR*. It is *those* meanings in which the classicist is trafficking even as he plies his strictly classical routes of reasoning. And it is for *that* very reason that the strictly classical moves such as *LEM* (or Dilemma) give expression to a realist metaphysical conception of the *relationship between* graspable propositional contents and the World. It is the *anti-realist’s* instantiating contents *P* for which the realist’s assertion of instances  $P \vee \neg P$  of *LEM* is, synthetically for the realist, telling us something about the World. In propounding the determinacy of reality, the realist is not investing the logical operators  $\neg$  and  $\vee$  with any further ‘classical’ ingredients of meaning. Rather, the realist is making a synthetic *a priori* claim *about Reality itself*, appropriately aided thereto by the anti-realistically licit meanings bestowed on the logical operators by the workings of the *other* logical rules besides the classical negation rules. Thus, it would be philosophically more insightful to parse this last phrase as “classical (negation rules)” rather than as “(classical negation) rules”. The ‘classical (negation rules)’ (and all the other strictly classical compound moves to which they give rise) do not reveal in, or exploit from, or confer upon the logical operators any extra, supposedly ‘classical’ ingredient of meaning. Rather,

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stants; so he is not really challenging the laws that we have always accepted and may therefore continue to accept. . . . [The intuitionist] denies that [the classical mathematician] has hold of a coherent meaning. . . . He acknowledges that he attaches meanings to mathematical terms different from those the classical mathematician ascribes to them; but he maintains that the classical meanings are incoherent and arise out of a misconception on the part of the classical mathematician about how mathematical language functions. Thus the answer to the question how it is possible to call a basic logical law in doubt is that, underlying the disagreement about logic, there is a yet more fundamental disagreement about the correct model of meaning, that is, about what we should regard as constituting an understanding of a statement.

applications of strictly classical rules make only grand metaphysical claims *about the determinacy of reality*, and indeed by virtue of having only such meanings as are recognized by the anti-realist, and adequately codified in *IR*.

We have now attained a new state of understanding of the realist's deductive ways. Much of his deductive meandering is based on meaning (as is that of the anti-realist); but a separable and easily identifiable part of the realist's why's and wherefore's is based on (or gives expression to) a strictly *metaphysical* conception of how things determinately are. It is this metaphysical conception that clinches the dichotomous nature of his logic. One must assume that, if disabused of that conception, he would refrain from giving it expression in his reasoning. He would cut back to using only the canon of *IR*. And, in doing so, he would be using the logical operators with precisely those meanings that they had borne all along. It was those meanings *plus* the now relinquished metaphysical conception that inflated the canon so as to include strictly classical moves. Those moves were based on a realist metaphysics, not on 'classical meanings'.

## 12 What if ...?

It might be amusing and instructive to ask "What if the neo-Dummettian considerations above had been available to Kant? How might subsequent philosophical developments have turned out?" Let us call the characters thus influenced Kant\*, Frege\*, etc. We are to imagine a rather prescient contemporary of Kant\*, called Mikhail Dommatt, who teaches Kant\* a few of the things available to Dummett.

My tongue-in-cheek conjecture is that Kant\* would still have insisted on the importance of the synthetic *a priori*. He\* would have included *LEM* alongside real analysis and Euclidean geometry. He would have been excited to learn from Dommatt about advances, since Aristotle, in propositional logic. Dommatt could have imparted to him the Gentzen-style rules of natural deduction for (intuitionistic) propositional logic, without teaching Kant\* anything about quantifiers.

Frege\* would in due course have sought to reduce what we now think of as Heyting arithmetic to his\* (intuitionistic) class theory. And Frege\*'s analysis of quantification, and discovery of *first-order* (intuitionistic) logic, would have earned him\* his\* influential place in analytical\* philosophy.

Russell\* would still have discovered his\* paradox in Frege\*'s system, for its proof is entirely intuitionistic. (Frege\*, by the way, might also have pointed out that his\* method of reduction, if successful, could be extended to reduce classical arithmetic to classical class theory, tinged though both would be by the synthetic *a priori* status of *LEM*.) Russell\* would have furnished his\* theory of types, in both intuitionistic<sup>34</sup> and classical forms, in order to avoid the paradox in Frege\*'s system.

The early Wittgenstein\* would have written his *Tractatus Logico-Philosophicus\** under Russell\*'s direction, but would have had to invent possible world semantics instead of the relatively trivial truth tables, thereby earning himself a much more impressive reputation as a logician. The later Wittgenstein\* would not have written anything, having been depressed to find that all his best ideas had been anticipated in the work of a relatively obscure contemporary of Kant\*'s by the name of Mikhail Dommat. Ludwig\* would have stayed in Norway, and bought it.<sup>35</sup>

The Logical Positivists\*, in rejecting all forms of the synthetic *a priori*, would have had to reject that most famous form: the Law of Excluded Middle. *Weg mit der Metaphysik!* Carnap\* could still have written in his notes for his Bauhaus lecture back in the late 1920\*'s. Popper\* would have claimed that there was no such thing as induction, and that the only important feature of scientific theories was their falsifiability. He\* would have pointed out, quite correctly, that *LEM* is superfluous, and does not increase our ability to subject scientific theories to the strongest possible tests. But as the relations of confirmation and support between evidence and hypotheses were paid more attention (*pace* Popper\*), Logical Positivism\* would in due course have become *anti-realist Logical Empiricism*.

Gödel\* would have shown (within an intuitionistic metatheory) the undecidability and essential incompleteness of both Heyting arithmetic and classical (Peano-Dedekind) arithmetic. Tarski\* would have put forward his\* justly celebrated account of the concept of truth in formalized languages, also using only an intuitionistic metatheory. He\* would have shown that his\* theory of truth met his\* adequacy condition without ever having to appeal to *LEM*. He\* would also have shown that the notion of arithmetical truth is not arithmetically definable (and he\* would have added a footnote

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<sup>34</sup>thereby making Martin-Löf\* somewhat superfluous for this brief history\*.

<sup>35</sup>Possible worlds semantics having been invented by the early Wittgenstein\*, and there being no later Wittgenstein\* writing about rule-following, the character Kripke\*, given prevailing ignorance (in the period 1960\*-1980\*) of the works of Mikhail Dommat, did not need to be invented.

to the effect that this result could not be circumvented by ‘going classical’ with one’s arithmetical theory).

A precocious and enterprising young American called Quine\* would have come to Prague in the early 1930\*'s to visit the already famous Carnap\*; and would in due course have presided over the merging of the American pragmatist tradition with that of the Logical Positivists\*. He would have written an influential logic text called *Methods of Logic*\*, in which (so some\* would later say) he\* regrettably over-emphasized intuitionistic theoremhood at the expense of intuitionistic deducibility. But many would have been dazzled by his thesis of the Radical Indeterminacy of Translation. His masterwork, *Word and Object*\*, would have ‘established’ such indeterminacy even while conceding that a principle of charity required us to read our own intuitionistic logic into the language of the jungle-dwellers. Quine\* would also have been credited with the acid observation that the small minority of logicians who wanted to use *LEM* ran the risk of ‘changing the subject matter’.

Church\* would have collapsed Ayer\*'s criterion of cognitive significance in his\* *JSL* review of *Language, Truth and Logic*\*, using only intuitionistic logic to do so. There would have been further attempts to provide a criterion, notably one from Carnap\* in 1956\*, after which the mood of pessimism would be summarized by Hempel\* in an influential article entitled ‘Problems and Changes in the Empiricist\* Criterion of Cognitive Significance’.

As the Gödel\*-Tarski\* results sank in, philosophers\* would have tempered their rejection of synthetic *a priori* status for all truths of arithmetic. Perhaps, they\* would think, the independent Gödel sentences, at least, were synthetic: for, in their proofs (when formalized at the meta-level) we\* need recourse to a truth predicate, occurring in new instances of the axiom schema of induction; yet there is no occurrence of such a predicate in the independent sentence itself; so how, then, could its truth be grasped on the basis of its meaning alone? But they\* would not have re-considered their\* eschewal of Bivalence and the Law of Excluded Middle, since by now they\* would have been convinced that *LEM* was epistemologically otiose.

Then, as the emphasis in American universities reverted to the history of 18\*th and 19\*th century philosophy\*, the works of the little-known Mikhail Dommatt would have come to light. Philosophers\* would have discovered therein the antidote required for the meaning skepticism of Quine\*. The analytic/synthetic distinction would have been re-instated, and new attempts made to explicate the important notion of cognitive significance.

As it happens, such developments\* would culminate in more or less the view that I think is correct now. And we would have reached it even after

having sent Michael Dummett in a Tardus back to Königsberg in 1775. Why, then: it *must* be correct!<sup>36</sup>

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<sup>36</sup>I apologize for not having pinned down the details for Cantor\*, Zermelo\*, Fraenkel\*, Brouwer\*, Skolem\* and Turing\*. They\* will have to be relegated to a sequel. Hegel\*, though, is otiose, given either Brandom or Brandom\*.