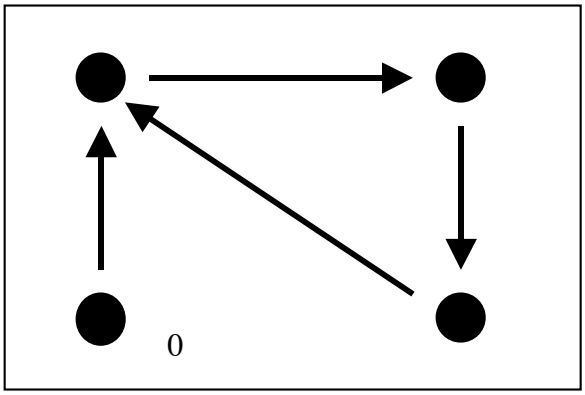
Exercise 8 in *Natural Logic*, on p.186.

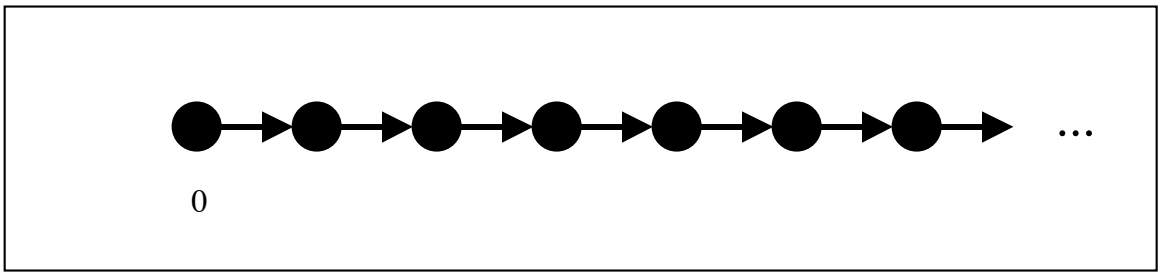
Find both finite and infinite counterexamples to the following invalid argument:

$$\forall x \forall y(Fxy \rightarrow \neg Fyx)$$
$$\forall x \exists y Fxy$$
$$\underline{\forall x \exists y \neg Fyx}$$
$$\forall x \exists y(Fyx \wedge \neg Fyy)$$

A finite counterexample is as follows.



An infinite counterexample can be obtained from the positive integers, with *Fxy* interpreted as '*x* immediately precedes *y*':



In each counterexample, we have

$\forall x \forall y (Fxy \rightarrow \neg Fyx)$	is true: arrows are one-way
$\forall x \exists y Fxy$	is true: every dot sends out an arrow
$\underline{\forall x \exists y \neg Fyx}$	is true: there is no ‘arrow-head cluster’

but

$\forall x \exists y (Fyx \wedge \neg Fyy)$	is false: the ‘initial element’ 0 receives no arrow