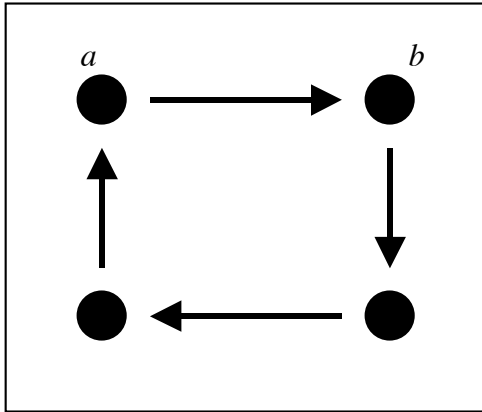


**Exercise 7 in *Natural Logic*, on p.185.**

Describe counterexamples to the following invalid arguments:

$$\frac{\forall x \exists y Rxy \quad \forall x (\exists y Rxy \rightarrow \exists z Rzx)}{\forall x \forall y \exists z (Rxz \wedge Rzy)}$$



Everyone respects someone. That is,  $\forall x \exists y Rxy$  is true.

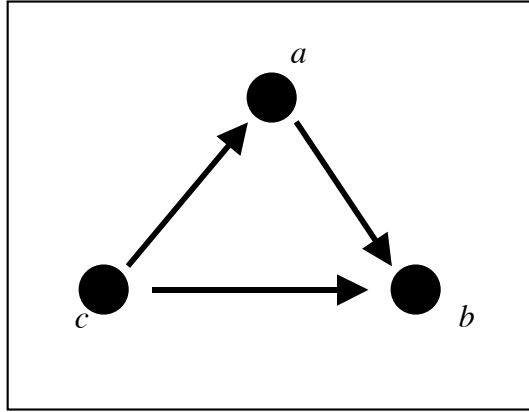
Everyone who respects someone is respected by someone. That is,  $\forall x (\exists y Rxy \rightarrow \exists z Rzx)$  is true.

But: no one whom  $a$  respects respects  $b$ ;

i.e.,  $\exists z (Raz \wedge Rzb)$  is false.

So  $\forall x \forall y \exists z (Rxz \wedge Rzy)$  is false.

$$\frac{\forall x \exists y (Rxy \vee Ryx) \quad \forall x \forall y (Rxy \rightarrow \forall z (Ryz \rightarrow Rxz))}{\exists x (\forall y Rxy \vee \forall y Ryx)}$$



The first premise says that every dot sends out or receives an arrow—which is true.

The second premise says that the arrow relation is transitive—which is true.

The truth of  $\forall y Ray \vee \forall y Rya$  requires the truth of  $Raa$ .  
 The truth of  $\forall y Rby \vee \forall y Ryb$  requires the truth of  $Rbb$ .  
 The truth of  $\forall y Rcy \vee \forall y Ryc$  requires the truth of  $Rcc$ .  
 But we have  $\neg Raa$ ,  $\neg Rbb$ , and  $\neg Rcc$ .  
 So  $\exists x (\forall y Rxy \vee \forall y Ryx)$  is false.