Exercise 2 in *Natural Logic*, on p.185.

Write down a sentence of first order logic with identity which is true of all and only queues (finite or infinite), using only the predicates

Px (*x* is a person),
Bxy (*x* is behind *y*) and
Cxy (*x* is a companion of *y*).

You may assume that only companions can occupy the same position in a queue.

We try to state some truths (i.e. formal sentences) about the situation, using the primitive vocabulary available. Our answer to the question will be the conjunction of all the formal sentences that we state. The hope is that any truth about queues (expressible in the available vocabulary) will follow logically from this conjunction.

Since the only 'things in the picture' are people, we can state at the outset

$$\forall x Px.$$

Everything else has to be stated in terms of the relations Bxy and Cxy.

The intuitive picture is that the places in a queue are *linearly* and *discretely* ordered. There is a first place; a second place; and so on. Every place other than the first has one immediately before it, and one immediately behind it.

We do not have the predicate 'x is a place', nor the predicate 'x is at place y (in the queue)'. We have to make do with the relations Bxy and Cxy.

In each place in a queue there may be just one person, or else a group of companions. So no-one is behind a companion:

$$\forall x \forall y (Cxy \rightarrow \neg Bxy).$$

The companionship relation is symmetric:

$$\forall x \forall y (Cxy \rightarrow Cyx).$$

Every queue has a head, i.e. someone who is 'first in line', or *behind no one*. Such a person may well have companions, who will also be first in line.

$$\exists x \neg \exists y Bxy$$

Since the queue may be finite or infinite, there may not be anyone 'last in line'. But everyone who is not first in line (that is, who is behind someone) will be *immediately behind* someone (that is, behind them *and with no-one in between*).

$$\forall y(\exists z Byz \rightarrow \exists w(Byw \wedge \neg \exists v(Bvw \wedge Byv)))$$

Behindness is *transitive*:

$$\forall x \forall y \forall z((Bxy \wedge Byz) \rightarrow Bxz)$$

and *asymmetric*:

$$\forall x \forall y(Bxy \rightarrow \neg Byx)$$

Companions occupy the same position in line:

$$\forall x \forall y(Cxy \rightarrow (\neg Bxy \wedge \neg Byx))$$

and people occupying the same position in line are companions:

$$\forall x \forall y((\neg Bxy \wedge \neg Byx) \rightarrow Cxy)$$

Companions are ‘behindness-indiscernible’, that is, *are behind* exactly the same people, and have exactly the same people *behind them*:

$$\forall x \forall y(Cxy \rightarrow \forall z(Bxz \rightarrow Byz))$$
$$\forall x \forall y(Cxy \rightarrow \forall z(Bzx \rightarrow Bzy))$$

Of any two non-companions, one is behind the other:

$$\forall x \forall y(\neg Cxy \rightarrow (Bxy \vee Byx))$$

In summary, our answer is the conjunction of the following sentences, here grouped by their use of primitives:

$$\forall xPx$$

$$\exists x\neg\exists yBxy$$
$$\forall y(\exists zByz\rightarrow\exists w(Byw\wedge\neg\exists v(Bvw\wedge Byv)))$$
$$\forall x\forall y\forall z((Bxy\wedge Byz)\rightarrow Bxz)$$
$$\forall x\forall y(Bxy\rightarrow\neg Byx)$$

$$\forall x\forall y(Cxy\rightarrow Cyx)$$

$$\forall x\forall y(Cxy\rightarrow\forall z(Bxz\rightarrow Byz))$$
$$\forall x\forall y(Cxy\rightarrow\forall z(Bzx\rightarrow Bzy))$$
$$\forall x\forall y(Cxy\rightarrow\neg Bxy)$$
$$\forall x\forall y(Cxy\rightarrow(\neg Bxy\wedge\neg Byx))$$
$$\forall x\forall y((\neg Bxy\wedge\neg Byx)\rightarrow Cxy)$$
$$\forall x\forall y(\neg Cxy\rightarrow(Bxy \vee Byx))$$

(Unfortunately, there is no way, using a first-order language, to stipulate that everyone has only finitely many people ahead of them in the queue. Nor is there any way to stipulate that at every place in the queue there are only finitely many people!)