

**Exercise 1 in *Natural Logic*, on p.185.**

1. 'The logical form of a sentence is best understood by considering the order in which it is built up from simpler expressions.' Illustrate this with respect to

(1) Only if it is the case that only if it rains only if it pours does it hail is it the case that it snows.

(2) Only boys loved only by girls love only girls.

(3) Only those boys loved only by girls love only girls.

(4) All boys loved by all girls love all girls.

(1) Only if it is the case that [only if it rains only if it pours does it hail] is it the case that it snows.

This has the overall form  $S \rightarrow [ \dots ]$ .

The material ‘...’ between the brackets is

‘only if (it rains only if it pours) does it hail’.

This has the overall form  $H \rightarrow ( \dots )$ .

The material ‘---’ between the parentheses is

‘it rains only if it pours’.

This has the form  $R \rightarrow P$ .

So the final logical form is  $S \rightarrow [ H \rightarrow ( R \rightarrow P ) ]$ .

(2) Only boys loved only by girls love only girls.

This has the overall form ‘Only  $\varphi$ s love  $\psi$ s’, i.e.  
 $\forall x(\psi x \rightarrow \varphi x)$ ,

where  $\varphi x$  :  $x$  is a boy loved only by girls;  
 $\psi x$  :  $x$  loves only girls.

Further details:

$\varphi x$  :  $x$  is a boy and only girls love  $x$ ; i.e.,  
 $x$  is a boy and everyone who loves  $x$  is a girl; i.e.,  
 $Bx \wedge \forall y(Lyx \rightarrow Gy)$ .

$\psi x$  : everyone whom  $x$  loves is a girl; i.e.  
 $\forall z(Lxz \rightarrow Gz)$ .

So the final logical form is

$$\forall x(\forall z(Lxz \rightarrow Gz) \rightarrow (Bx \wedge \forall y(Lyx \rightarrow Gy))).$$

(3) Only those boys loved only by girls love only girls.

This says

*‘Of the boys, only those who are loved only by girls love only girls’.*

Equivalently:

*‘Of the boys, those who love only girls are loved only by girls’.*

This has the overall form

for all  $x$ ,  
if  $x$  is a boy,  
then:  
if  $x$  loves only girls,  
then  $x$  is loved only by girls

In formal notation:

$$\forall x(Bx \rightarrow (\forall y(Lxy \rightarrow Gy) \rightarrow \forall z(Lzx \rightarrow Gz))).$$

(4) All boys loved by all girls love all girls.

This has the overall form 'All  $\varphi x \psi$ ', i.e.  $\forall x(\varphi x \rightarrow \psi x)$

where

$\varphi x$  :  $x$  is a boy loved by all girls;

$\psi x$  :  $x$  loves all girls.

Further details are as follows.

$\varphi x$  :  $x$  is a boy and  $x$  is loved by all girls; i.e.

$x$  is a boy and all girls love  $x$ ; i.e.

$Bx \wedge \forall y(Gy \rightarrow Lyx)$ .

$\psi x$  : every girl is loved by  $x$ ; i.e.

$\forall z(Gz \rightarrow Lxz)$ .

So the final logical form is

$\forall x((Bx \wedge \forall y(Gy \rightarrow Lyx)) \rightarrow \forall z(Gz \rightarrow Lxz))$ .