PHIL 250: Exercise 1 Model Answers

Let the *extra-logical* vocabulary of the language **L** consist of:
exactly two names a, b ;
exactly two one-place predicates F, G ; and
exactly two two-place predicate L, R.
The language **L** contains no other extra-logical vocabulary.

The *logical* vocabulary of **L** consists of:
the negation sign $\neg$;
the signs $\wedge$, $\vee$, $\rightarrow$ for conjunction, disjunction and implication;
the existential and universal quantifiers $\exists$, $\forall$; and
the two-place relation = of identity.

We use the following 'translation manual' for the primitive extra-logical expressions:

a	:	Albert
b	:	Betty
Fx	:	x is flatulent
Gx	:	x is grumpy
Lxy	:	x loves y
Rxy	:	x is richer than y

Translate the following English sentences into the formal notation of **L**:

1. Albert is flatulent and loves Betty.
   $Fa \wedge Lab$
2. Albert isn't flatulent but loves Betty.
   $\neg Fa \wedge Lab$
3. Betty is grumpy but Albert loves her.
   $Gb \wedge Lab$
4. If Betty is flatulent then Albert loves her.
   $Fb \rightarrow Lab$
5. If Albert is flatulent then Betty doesn't love him.
   $Fa \rightarrow \neg Lba$
6. Either Albert is flatulent or Betty is grumpy and loves him.
   $Fa \vee (Gb \wedge Lba)$
7. Albert is flatulent and grumpy only if Betty doesn't love him.
   $(Fa \wedge Ga) \rightarrow \neg Lba$
8. If Albert is either flatulent or grumpy then he doesn't loves Betty.
   $(Fa \vee Ga) \rightarrow \neg Lab$

9. Albert is flatulent but richer than Betty, and she is grumpy but loves him.
   $(Fa \wedge Rab) \wedge (Gb \wedge Lba)$
10. Someone is grumpy and richer than Betty but no one loves him.
    $\exists x((Gx \wedge Rxb) \wedge \neg \exists y Lyx)$
11. Everyone richer than Betty loves himself.
    $\forall x(Rxb \rightarrow Lxx)$
12. No one who is flatulent loves Betty.
    $\neg \exists x(Fx \wedge Lxb)$
13. Someone who is flatulent is richer than Betty and loved by her.
    $\exists x(Fx \wedge (Rxb \wedge Lbx))$
14. Everyone whom Betty loves is grumpy if flatulent.
    $\forall x(Lbx \rightarrow (Fx \rightarrow Gx))$
15. Everyone richer than Betty is either grumpy or flatulent.
    $\forall x(Rxb \rightarrow (Gx \vee Fx))$
16. Someone loves Betty and is richer than Albert, who is flatulent.
    $\exists x(Lxb \wedge Rxa) \wedge Fa$
17. Everyone richer than Albert loves everyone whom he loves.
    $\forall x(Rxa \rightarrow \forall y(Lay \rightarrow Lxy))$
18. Anyone richer than Albert loves no one whom he loves.
    $\forall x(Rxa \rightarrow \neg \exists y(Lay \wedge Lxy))$
19. Anyone grumpy is loved by no one but Betty.
    i.e. Anyone grumply is loved by Betty and no one other than Betty loves him:
    $\forall x(Gx \rightarrow (Lbx \wedge \neg \exists y(\neg y=b \wedge Lyx)))$
    Equivalently:
    Anyone grumpy is loved by Betty and anyone who loves him is identical to Betty
    $\forall x(Gx \rightarrow (Lbx \wedge \forall y(Lyx \rightarrow y=b)))$; or
    Anyone grumpy is loved by, and only by, Betty
    $\forall x(Gx \rightarrow \forall y(Lyx \leftrightarrow y=b))$
20. Anyone richer than Albert is loved by everyone poorer than Betty.
    i.e., Anyone richer than Albert is loved by everyone than whom Betty is richer
    $\forall x(Rxa \rightarrow \forall y(Rby \rightarrow Lyx))$

Translate the following sentences of **L** into English:

1. $Ga \wedge Fb$
   Albert is grumpy and Betty is flatulent
2. $Rab \wedge \neg Fa$
   Albert is richer than Betty and is not flatulent
3. $Laa \rightarrow Lbb$
   If Albert loves himself then Betty loves herself
4. $\forall x Fx$

Everyone is flatulent

5. $\exists x Gx$
   Someone is grumpy
6. $\neg \forall x Fx$
   Not everyone is flatulent
7. $\neg \exists x Gx$
   No one is grumpy
8. $\exists x \neg Gx$
   Someone is not grumpy
9. $\forall x \neg Fx$
   Everyone is non-flatulent
10. $\forall x \neg Lxx$
    Everyone does not love him/herself
11. $\forall x \forall y Lxy$
    Everyone loves everyone
12. $\forall x \exists y \neg Lyx$
    Everyone is unloved by someone
13. $\exists x \exists y Rxy$
    Someone is richer than someone
14. $\forall x (Lbx \rightarrow Fx)$
    Everyone Betty loves is flatulent
15. $\exists x (Rbx \vee Fx)$
    Someone is poorer than Betty or flatulent
    (which is better, stylistically, than:
    Someone is such that Betty is richer than him and he is flatulent)
16. $\forall x (\neg a = x \rightarrow Lxb)$
    Everyone other than Albert loves Betty
17. $\neg \exists x (Fx \wedge Gx \wedge \forall y Lyx)$
    No one is flatulent and grumply and loved by everyone
18. $\exists x Lbx \wedge \neg \exists x Lax$
    Someone is loved by Betty but Albert loves no one
19. $\exists x (Lbx \wedge \neg Lax)$
    Someone Betty loves is not loved by Albert
20. $\forall x \exists y (Lxy \wedge Lyx)$
    Everyone loves, and is loved by, someone