

Model answers to the deductive exercises.

1.

$$\frac{\frac{\forall xFx}{Fa}}{\exists xFx}$$

2.

$$\frac{\frac{\frac{\forall xFx}{Fa \quad \neg Fa} \quad \perp}{\neg \neg Fa} \quad (1)}{\forall x \neg \neg Fx}$$

3.

$$\frac{\frac{\forall x \neg \neg Fx}{\neg \neg Fa} \quad (DNE)}{\forall x Fx}$$

$$\begin{array}{c}
4. \quad \frac{\frac{\frac{}{(2)} \text{Fa}}{\text{Fa}} \quad \frac{}{(1)} \neg \text{Fa}}{\neg \neg \text{Fa}}}{\perp} (1) \\
\frac{\frac{\exists x Fx}{\exists x \neg \neg Fx} \quad \frac{\exists x \neg \neg Fx}{\exists x \neg \neg Fx} (2)}{\exists x \neg \neg Fx}
\end{array}$$

$$\begin{array}{c}
5. \quad \frac{\frac{\frac{}{(2)} \neg \neg \text{Fa}}{\neg \neg \text{Fa}} \quad \frac{}{(1)} \neg \text{Fa}}{\perp} (1)}{\text{Fa}} \text{(CR)} \\
\frac{\frac{\exists x \neg \neg Fx}{\exists x Fx} \quad \frac{\exists x Fx}{\exists x Fx} (2)}{\exists x Fx}
\end{array}$$

$$\begin{array}{c}
6. \quad \frac{\frac{\frac{}{(1)} \text{Fa} \wedge \text{Ga}}{\text{Fa}}}{\exists x (Fx \wedge Gx)} \quad \frac{\exists x (Fx \wedge Gx)}{\exists x Fx} (1)}{\exists x Fx}
\end{array}$$

$$\begin{array}{c}
7. \quad \frac{\frac{\frac{\forall x (Fx \wedge Gx)}{\text{Fa} \wedge \text{Ga}}}{\text{Fa}}}{\forall x Fx}
\end{array}$$

12.

$$\begin{array}{c} \frac{\forall x(Fx \rightarrow Gx)}{Fa \rightarrow Ga} \quad \frac{}{Fa} \quad \frac{}{\neg Ga} \\ \frac{}{Ga} \quad \frac{}{\neg Fa} \\ \frac{}{\neg Ga \rightarrow \neg Fa} \\ \forall x(\neg Gx \rightarrow \neg Fx) \end{array}$$

13.

$$\begin{array}{c} \frac{}{\forall yLay} \\ \frac{}{Lab} \\ \frac{\exists x\forall yLxy \quad \exists xLxb}{\exists xLxb} \\ \forall y\exists xLxy \end{array}$$

14.

$$\begin{array}{c} \forall x\forall yLxy \\ \forall yLay \\ \frac{}{Laa} \\ \forall xLxx \end{array}$$

15.

$$\begin{array}{c}
 \frac{}{\text{Laa}}^{(1)} \\
 \frac{}{\exists y \text{Lay}} \\
 \frac{\exists x \text{Lxx} \quad \exists x \exists y \text{Lxy}}{\exists x \exists y \text{Lxy}}^{(1)}
 \end{array}$$

16.

$$\begin{array}{c}
 \frac{}{\forall x \text{Fx}}^{(1)} \quad \frac{}{\forall x (\text{Fx} \rightarrow \text{Gx})} \\
 \frac{\text{Fa} \quad \text{Fa} \rightarrow \text{Ga}}{\text{Ga}} \\
 \frac{}{\forall x \text{Gx}}^{(1)} \\
 \forall x \text{Fx} \rightarrow \forall x \text{Gx}
 \end{array}$$

17.

$$\begin{array}{c}
 \frac{}{\text{Fb}}^{(3)} \quad \frac{}{\text{Ga}}^{(1)} \quad \frac{}{\text{Fb}}^{(2)} \quad \frac{}{\neg \text{Fb}}^{(3)} \\
 \frac{\exists x \text{Fx} \quad \exists x \text{Fx} \rightarrow \exists x \text{Gx}}{\exists x \text{Gx}} \quad \frac{\text{Fa} \rightarrow \text{Ga}}{\exists x (\text{Fx} \rightarrow \text{Gx})}^{(1)} \quad \frac{\perp}{\text{Gb}}^{(2)} \\
 \frac{\exists x \text{Gx} \quad \exists x (\text{Fx} \rightarrow \text{Gx})}{\exists x (\text{Fx} \rightarrow \text{Gx})}^{(1)} \quad \frac{\text{Fb} \rightarrow \text{Gb}}{\exists x (\text{Fx} \rightarrow \text{Gx})}^{(3)} \\
 \text{(Dilemma)} \quad \exists x (\text{Fx} \rightarrow \text{Gx})
 \end{array}$$

18.

$$\begin{array}{c}
 \frac{}{\text{Fa} \wedge \neg \text{Ga}}^{(1)} \quad \frac{}{\forall x(\text{Fx} \rightarrow \text{Gx})} \\
 \frac{\text{Fa} \wedge \neg \text{Ga}}{\text{Fa}} \quad \frac{\forall x(\text{Fx} \rightarrow \text{Gx})}{\text{Fa} \rightarrow \text{Ga}} \quad \frac{}{\text{Fa} \wedge \neg \text{Ga}}^{(1)} \\
 \frac{}{\text{Ga}} \quad \frac{}{\neg \text{Ga}} \\
 \frac{\exists x(\text{Fx} \wedge \neg \text{Gx})}{\perp}^{(1)} \\
 \frac{\perp}{\neg \forall x(\text{Fx} \rightarrow \text{Gx})}^{(2)}
 \end{array}$$

19.

$$\begin{array}{c}
 \frac{}{\forall x \text{Fx}}^{(2)} \quad \frac{}{\forall x(\text{Fx} \rightarrow \text{Gx})} \\
 \frac{\forall x \text{Fx}}{\text{Fa}} \quad \frac{\forall x(\text{Fx} \rightarrow \text{Gx})}{\text{Fa} \rightarrow \text{Ga}} \quad \frac{}{\neg \text{Ga}}^{(1)} \\
 \frac{}{\text{Ga}} \quad \frac{}{\neg \text{Ga}} \\
 \frac{\exists x \neg \text{Gx}}{\perp}^{(1)} \\
 \frac{\perp}{\neg \forall x \text{Fx}}^{(2)}
 \end{array}$$

20.

$$\begin{array}{c}
 \frac{}{\forall x(\text{Fx} \rightarrow \text{Gx})} \\
 \frac{}{\text{Fa}}^{(1)} \quad \frac{\forall x(\text{Fx} \rightarrow \text{Gx})}{\text{Fa} \rightarrow \text{Ga}} \\
 \frac{}{\text{Ga}} \\
 \frac{}{\exists x \text{Gx}} \quad \frac{}{\neg \exists x \text{Gx}} \\
 \frac{}{\perp}^{(1)} \\
 \frac{}{\neg \text{Fa}} \\
 \frac{}{\forall x \neg \text{Fx}}
 \end{array}$$

25.

$$\frac{\frac{\text{---}(1)}{\neg Fa}}{\exists x \neg Fx} \quad \frac{\text{---}(2)}{\neg \exists x \neg Fx}}{\underline{\exists x \neg Fx} \quad \underline{\neg \exists x \neg Fx}}$$

$$\frac{\text{(CR)} \frac{\perp}{\text{---}(1)}}{Fa}}{\underline{\forall x Fx} \quad \underline{\neg \forall x Fx}}$$

$$\text{(CR)} \frac{\perp}{\text{---}(2)} \\ \exists x \neg Fx$$
