In all the problems solved below, the premises of the argument in question are listed first, and the conclusion is listed last.

Each argument is invalid. With each argument, the task is to describe a model that makes its premise(s) true but that makes its conclusion false. ***This is what it takes to establish the invalidity of the argument.*** For the existence of such a 'counterexemplary' model is what the invalidity of the argument consists in.

The following convention will be observed for the metalinguistic description of finite models. The individuals in the domain will be exhaustively listed, as *a, b, c,* … etc. as needed, and they will be assumed to be mutually distinct. Then the *positive* atomic facts will be listed, by means of the appropriate atomic sentences of the metalanguage, formed from predicates and individuals-as-names. (The individual *a*, say, will be named by the same *name a*, this name being assumed available in the metalanguage. We can say that individuals name themselves; in the metalanguage, they are their own names.) If, say, *a* and *b* are among the individuals listed, and the atomic sentence *Lab* does *not* occur in the list given, then it is to be assumed that the relation *L* does *not* obtain between the individuals *a* and *b* (in that order).

**Example 1.** Here is the way we shall describe the model in which *a* and *b* are the only two individuals; these two individuals ***L***ove one another but not themselves; and *a* but not *b* is ***F***at:

*a, b; Lab; Lba; Fa*

Note that the absence of the sentences *Fb*, *Laa* and *Lbb* from this list indicates that these sentences are *false*, and therefore that their *negations* $\neg Fb$, $\neg Laa$ and $\neg Lbb$ are *true*. We *could* have written, more exhaustively:

*a, b; Lab; Lba; Fa;* $\neg Fb$; $\neg Laa$;$\neg Lbb$.

but there is no need to. For the governing convention is that *absence entails falsity*. (This is what the logic-programming community calls the 'closed world assumption': if the proposition *P* does not appear in or follow from what is given, then *P* is false, and its negation, $\neg P$, holds.)

The more exhaustive list just given, which includes negated atomic sentences to register explicitly the atomic falsehoods, is called the *atomic diagram* of the model. Our descriptive convention is to display all and only that part of the atomic diagram that does not involve the negation sign. The whole atomic diagram can then be constructed in the obvious way.

**Example 2.** Here is the way we shall describe the model in which *a* and *b* are the only two individuals; these two individuals are fat and love themselves, and love only themselves:

*a, b; Fa; Fb; Laa; Lbb.*

The full atomic diagram would be:

*a, b; Fa; Fb; Laa; Lbb; ¬Lab; ¬Lba.*

With this descriptive (non-diagrammatic) convention in place, we now provide for each invalid argument a (description of a) model that would make the argument's premise(s) true but its conclusion false.

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1.

$\exists x Fx$ Premise
$\forall x Fx$ Conclusion

*a, b; Fa.* Description of model

***The premise of the argument is true in the model described:***

The following model-relative 'evaluation-proof' shows that the premise $\exists x Fx$ is true:

$$\frac{Fa}{\exists x Fx}$$

Note that the atomic 'axiom' Fa is permitted because *Fa* is true in the model given (as one can tell from the presence of *Fa* in the description of the model).

***The conclusion of the argument is false in the model described:***

The following model-relative 'evaluation-disproof' shows that the conclusion $\forall x Fx$ is false:

$$\frac{\dfrac{\forall x Fx}{Fb}}{\bot}$$

Note that the atomic inference from Fb to ⊥ is permitted because Fb is false in the model given (as one can tell from the absence of *Fb* from the description of the model).

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The foregoing solution to Problem 1 illustrates the method of model-relative proof of the premises, and model-relative disproof of the conclusion, of an invalid argument. For the remaining solutions we omit the fine details of such proofs and disproofs, and provide only model-descriptions relative to which the student will easily construct the proofs and disproofs that are required for the problem at hand.

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2.

∀xFx
∀x¬Fx

*a; Fa.*

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3.

∀x¬Fx
∃xFx

*a.*

---

4.

∀xFx
∃x¬Fx

*a; Fa.*

---

5.

∃x¬¬Fx
∀xFx

*a, b; Fa.*

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6.

∀x(Fx∨Gx)
∀xFx

*a, b; Fa; Gb.*

7.

$\exists x(Fx \wedge Gx)$
$\forall x(Fx \vee Gx)$

*a, b; Fa; Ga.*

8.

$\exists x(Fx \vee Gx)$
$\exists xFx \wedge \exists xGx$

*a, b; Fa.*

9.

$\exists xFx \wedge \exists xGx$
$\exists x(Fx \wedge Gx)$

*a, b; Fa; Gb.*

10.

$\forall xFx$
$\forall x(Fx \wedge Gx)$

*a; Fa.*

11.

$\forall xFx \rightarrow \forall xGx$
$\forall x(Fx \rightarrow Gx)$

*a, b; Fa.*

12.

∀x(Fx→Gx)
∀x(Gx→Fx)

*a, b; Fa; Ga; Gb*.

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13.

∀y∃xLxy
∃x∀yLxy

*a, b; Laa; Lab*.

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14.

∀xLxx
∀x∀yLxy

*a, b; Laa; Lbb*.

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15.

∃x∃yLxy
∃xLxx

*a, b; Lab*.

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16.

∀x(Fx→Gx)
¬∀xFx → ¬∀xGx

*a, b; Fa; Ga; Gb*.

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17.

$\exists xFx \rightarrow \exists xGx$
$\forall xFx \rightarrow \forall xGx$

*a, b; Fa; Fb; Ga.*

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18.

$\exists x(Fx \wedge \neg Gx)$
$\forall x(Fx \rightarrow \neg Gx)$

*a, b; Fa; Fb; Gb.*

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19.

$\exists x \neg Gx$
$\forall x(Fx \rightarrow Gx)$
$\neg \exists xFx$

*a, b; Fa; Ga.*

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20.

$\exists x \neg Fx$
$\exists x(Fx \vee Gx)$
$\exists x \neg Gx$

*a, b; Ga; Gb.*

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21.

$\neg \forall xFx$
$\neg \exists xFx$

*a, b; Fa.*

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22.

∃x¬Fx
∀x¬Fx

*a, b; Fa.*

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23.

¬∀xFx
∀x¬Fx

*a, b; Fa.*

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24.

∃x¬Fx
¬∃xFx

*a, b; Fa.*

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