Stud. Hist. Phil. Sci. 34 (2003) 45–57

Studies in History
and Philosophy
of Science

www.elsevier.com/locate/shpsa

Reflections on Kant's concept (and intuition) of space

Lisa Shabel

Department of Philosophy, The Ohio State University, 230 North Oval Mall, 350 University Hall, Columbus, OH 43210, USA

Abstract

In this paper, I investigate an important aspect of Kant's theory of pure sensible intuition. I argue that, according to Kant, a pure concept of space warrants and constrains intuitions of finite regions of space. That is, an *a priori* conceptual representation of space provides a governing principle for all spatial construction, which is necessary for mathematical demonstration as Kant understood it.

Keywords: Kant; Space; Pure sensible intuition; Philosophy of mathematics

> No one can be in doubt concerning the meaning of the ground of the possibility of a pure sensible intuition, except he who roams through the *Critique* with the help of a dictionary but does not think it through. (Kant, *On a discovery according to which any new critique of pure reason has been made superfluous by an earlier one*)[1]

Despite what Kant himself alleged, the meaning of his theory of pure sensible intuition remains unsettled, and continues to be a topic of philosophical interpretation and debate. Our honoree, Gerd Buchdahl, evidently claimed that his own interest in philosophy could be traced to his first consideration of the difficult arguments of

E-mail address: shabel.1@osu.edu (L. Shabel).

[1] Allison (1983), p. 136. All subsequent quotations will follow the usual A/B convention and are from Kant (1998).

0039-3681/03/$ - see front matter © 2003 Elsevier Science Ltd. All rights reserved.
doi:10.1016/S0039-3681(02)00089-4

Kant's Transcendental Aesthetic, the science of all principles of *a priori* sensibility.[2] Proceeding without a dictionary, my goal here is to set out some ideas about how best to understand Kant's concept of space and, in particular, how best to understand the role that Kant took that concept to play in mathematical reasoning.[3]

It will help to begin with a description of how the position described herein fits into a larger interpretive picture I am developing, the ultimate goal of which is to provide an account of Kant's philosophy of mathematics and its role in his general epistemology. In a paper entitled 'Kant on the "symbolic construction" of mathematical concepts' (Shabel, 1998) I defended the view that, for Kant, all mathematical construction—even the 'symbolic construction' of algebra and analysis—is ostensive, and dependent on the spatial constructions of geometry. Since for Kant all mathematical reasoning is constructive, this result implies that, for Kant, *all mathematical reasoning is founded on cognition of space*. Understanding Kant's account of our cognition of space, then, is a necessary step in providing an account of his philosophy of mathematics.

But understanding Kant's account of our cognition of space of course requires a walk down the well trodden path of the Aesthetic. To reconsider Kant's account of our cognition of space as it is articulated in the arguments of the Aesthetic requires reconsidering Kant's account of both *the relation between the concept of space and the practice of geometry*, and the relation between the practice of geometry and our experience of the empirical world. The broad outline of Kant's account of these relations, as I interpret it, runs as follows. A concept of SPACE provides us with a 'principle' that governs our pure intuitions of individual spatial regions,[4] which it is the business of (pure Euclidean) geometry to construct and investigate; pure geometry in turn provides us with the means for structuring our (spatial) experience of empirical objects. I will here be concerned merely to begin exploring Kant's

[2] 'Gerd Buchdahl introduced himself to philosophy at the age of fifteen, when, 'with bated breath', he read Kant's arguments about space and time in his father's so far unopened Insel Verlag edition of the *Critique*' (Woolhouse, 1988, p. 1).

[3] One might suppose that this issue, at least, is well understood, given Michael Friedman's recent treatment in his *Kant and the exact sciences* (Friedman, 1992). There and in his article 'Geometry, construction and intuition in Kant and his successors' (Friedman, 2000), Friedman defends an interpretation of the Aesthetic according to which constructive geometric procedures serve, in some sense, to delimit the content of the original representation of space. See especially pp. 67 ff. of Friedman, 1992. I am here taking some initial steps toward a slightly different interpretation of the Aesthetic, to be completed elsewhere.

[4] The Kantian way to put this point would be to say that our concept of SPACE unifies the pure sensible manifold in a *formal intuition*, thus representing space as an object. See the infamous footnote at B160, as well as one less infamous at B136. In what follows I will emphasize Kant's distinction between the concept of SPACE and the construction of individual spaces represented as intuitions. In order to track this distinction, I will use 'SPACE' when I wish to designate the concept of space and 'space' or 'spaces' to designate intuitions of finite spatial regions. In what follows, I hope to articulate the sense in which SPACE is a 'principle' that governs the construction of spaces. (In what follows, I do *not* attempt to offer a complete account of such constructions, which would require discussion of the role of productive imagination in performing such constructions, as well as the axiomatic judgements made on the basis of such constructions.)

account of the first relation cited above, between the concept of SPACE and the practice of pure geometry.[5]

I take it that a partial account of this relation is, in some sense, the task of Kant's 'Metaphysical Exposition of the Concept of Space', which is itself the first task of the Transcendental Aesthetic. As already noted, the Aesthetic is meant to constitute 'a science of all principles of *a priori* sensibility' (A21/B35), and begins with an investigation of SPACE, which Kant rather unhelpfully identifies as one of 'two pure forms of sensible intuition as principles of *a priori* cognition' (A22/B36). In order to clarify Kant's task here, recall that *sensibility* is 'the capacity (receptivity) to acquire representations through the way in which we are affected by objects' (A19/B33); i.e., that cognitive capacity by which we *receive* representations of objects.[6] Famously, of course, Kant investigates the *form* of sensible representations apart from their *matter* since, he argues, the structure for the *a posteriori* representations we receive from sensation must itself be *a priori* (A20/B34). This leads him to the science of *a priori* sensibility, which suggests that our capacity to receive representations of objects (namely, sensibility) *includes* a capacity to receive representations of the *a priori* form of objects (namely, *a priori* sensibility). Since SPACE is one of two such *a priori* forms, *a priori* sensibility includes *a capacity to receive pure representations of space*.

Here the notion of 'receptivity' begins to seem a bit awkward and, possibly, misleading: while our receptivity to *empirical* sensible representations might be construed as depending on the structure provided by *pure* sensible representations (such as the givenness of an empirical spatial object, like the shape of a window, depending on a pure representation of a spatial region, like a square), it makes little explanatory sense to describe these *pure* sensible representations as themselves *received*. Otherwise, one would be left to wonder what sort of structure was in place for *their* reception. Moreover, such an explanation would make it difficult to account for a 'pure' sensible representation being both received (from where?) and constructed *a priori*, as Kant wishes to do. Now, Kant's ambiguous phrase 'capacity (receptivity)' begins to demand interpretation; with respect to SPACE, it makes more (Kantian) sense to speak of an *a priori* sensible capacity *to construct* (rather than receive) pure representations of space(s), which themselves provide us with an *a posteriori* sensible capacity *to receive* empirical representations of spatial objects.

Returning now to the relation between the concept of SPACE and the practice of geometry, it will be best to construe SPACE as providing us with a 'principle'[7]

[5] I thus attempt to provide here neither an interpretation of the second relation (between the practice of geometry and our experience of the empirical world) nor a complete interpretation of the first. I have explored these further issues in manuscripts entitled 'Kant's "argument from geometry"' (forthcoming in *The Journal of the History of Philosophy*) and 'Kant on the "axioms" of Euclid's geometry'.

[6] While it might seem strange to speak of a '*capacity* to *receive* representations' (since this could suggest a *passive power*), this is the gloss I choose to put on Kant's identification of sensibility as a 'capacity (receptivity) to acquire representations'. As we will see, our receptivity to *spatial* representations includes a capacity to *construct*, and so cannot be construed as entirely passive.

[7] Or rule, or condition. Kant uses the language of space as a sensible *principle* at A22/B36 and A619/B647.

governing *a priori* constructions of finite spatial regions, which become the objects of pure geometry.[8] This helps to make sense of Kant's claim that our concept of SPACE 'contains a pure intuition in itself... [and thus] can be constructed' (A719/B748): the *concept* of SPACE provides us with a 'principle' for performing constructions in pure *intuition*, in particular, *a priori* constructions of basic spatial regions (e.g., a particular line, a particular shape) that exhibit particular basic spatial concepts (e.g., straight line, closed plane figure). In other words, the construction of the basic spatial regions that become the objects of investigation for the pure geometer originates and proceeds in accordance with constructive warrants and constraints that are themselves determined by the content of the original concept of SPACE; codification of these warrants and constraints provides the basis for an *a priori* science of space. Thus does pure geometry, the science of space, result from the employment of *a priori* principles of spatial sensibility[9] in combination with distinctively intuitive mathematical reasoning to generate a systematic theory of spatial construction.

To put the point in even more Kantian terms, an originally given 'pure sensible *concept*' of SPACE seems to provide us with the 'ground of the possibility of a pure sensible *intuition*' of space, thus affording us an object for pure geometry. What I hope to do in what follows is to fill out some details of the foregoing by exploring the apparent ambiguity in Kant's explicit designation of SPACE[10] as both concept *and* intuition, as well as the role that *a priori* construction plays in clarifying this ambiguity.[11]

1.

Kant sets himself the task of providing a metaphysical exposition of the *concept* of SPACE:

> . . . we will expound the concept of space first. I understand by **exposition** (*expositio*) the distinct (even if not complete) representation of that which belongs to a concept; but the exposition is **metaphysical** when it contains that which exhibits the concept **as given** *a priori*. (A23/B38; emphasis original)

[8] The representations thus afforded by pure geometry find 'application' only upon being used to structure our experience of empirical objects (by, in particular, structuring our received empirical representations of objects): an account of this application would be an account of the second relation noted above, between the practice of geometry and our experience of the empirical world.

[9] Thus there are *a priori* (aesthetic) principles of pure sensibility akin to the *a priori* principles of pure understanding. In the case of space, these principles provide the axioms for a science of space, namely geometry; in the case of time, these principles provide the axioms for a science of time, namely mechanics [Ak 4:283].

[10] Here I mean to refer to both the concept of SPACE and the intuition of individual spaces, since I am identifying an ambiguity between the two.

[11] I will not here survey the literature on this issue; my strategy is simply to articulate what I take Kant's concept of SPACE to contain, in order to facilitate an interpretation of his philosophy of mathematics.

He proceeds to argue, famously, that 'the original representation of space is an ***a priori* intuition**, not **a concept**' (A25/B40). This is puzzling given Kant's own view that the elements of our cognition are divided exhaustively among concepts and intuitions: it is strange at best, and contradictory at worst, to suppose that we can understand the content of our *concept* of SPACE only by seeing that we in fact use intuitions, and not concepts, to represent that content. It is likewise puzzling that 'a science of all principles of *a priori* sensibility' (A21/B35) should begin with an exposition of a *concept* at all: the representations received by the faculty of sensibility are all and only intuitions, while those thought by the faculty of understanding are all and only concepts. Additionally,

> . . . these two faculties or capacities cannot exchange their functions. The understanding is not capable of intuiting anything, and the senses are not capable of thinking anything. Only from their unification can cognition arise. But on this account *one must not mix up their roles*, rather one has great cause to separate them carefully from each other and distinguish them. (A51-2/B75-6; emphasis added)

Given that the science of *sensibility* nevertheless begins with examination of a *concept*, it is worth asking what role there is for a *concept* to play in explaining the uniquely *intuitive* contribution that sensibility makes to our cognition of objects. In particular, it is worth asking what role the *concept* of SPACE plays in explaining the uniquely *intuitive* contribution that spatial sensibility makes to our cognition of objects.

We can gain some insight into these questions if we are willing to take seriously the idea that—despite his exhaustive classification of the understanding as the unique domain of concepts, and sensibility as that of intuitions—Kant nevertheless conceived an important role for pure sensible or *aesthetic* concepts. In a passage that serves to introduce the idea of the transcendental logic, Kant distinguishes the pure concepts of the understanding as those concepts that are 'of neither empirical nor aesthetic origin' (A57/B81), implying that both empirical and *aesthetic* concepts stand in contrast to the pure concepts of the understanding (as well as to each other). Here, he explicitly identifies the existence of *concepts* that are born of sensibility. Presumably, then, where Kant identifies SPACE as a *concept*, he means to identify it as a *pure aesthetic concept*.

Now, one might be tempted to suppose that Kant was merely careless to describe SPACE as an aesthetic *concept*, since he clearly argued that our representations of space are intuitive. On this supposition, employment of a concept of SPACE serves merely to designate that which can only be grasped intuitively. Or, one might suppose that an 'aesthetic concept' of SPACE designates a discursive function that gathers multiple thoughts of singular spaces (thoughts that would be 'empty' without intuitive content), failing to consider that a pure aesthetic concept of SPACE might play any further role in producing the content of such thoughts. On these suppositions, the concept of SPACE would be indistinguishable from what Kant calls 'the general concept of spaces in general' (A25/B39), presumably that concept 'which is common to a foot as well as an ell' (A25). But Kant explicitly states that such a general concept itself rests on limitations of space (A25/B39) and cannot itself be the source of the boundlessness of space (A25). Thus, an exposition of such a 'general concept of spaces in general' could not be expected to satisfy Kant's goals in the Transcendental Aesthetic.

Thus, these interpretive suggestions fail to make sense of Kant's identification of a concept of SPACE that is strictly identical neither to a general concept of spaces in general, nor to any particular intuition. My alternative suggestion is to suppose that Kant identifies a concept of SPACE in order to isolate a 'principle' of sensibility: a *sensible* 'rule', 'function' or 'unity'[12] that *warrants and constrains* our capacity to intuit space. Put another way, the aesthetic concept of SPACE *conditions* or *governs* our capacity to form intuitions of any and all particular spatial regions.[13]

This way of thinking about the concept of SPACE is illuminated by Kant's 'constructibility' claim, made in the Discipline of Pure Reason:

> Now an *a priori* concept (a non-empirical concept) either already *contains a pure intuition in itself, in which case it can be constructed*; or else it contains nothing but the synthesis of possible intuitions, which are not given *a priori*, in which case one can well judge synthetically and *a priori* by its means but only discursively, in accordance with concepts, and never intuitively through the construction of the concept. (A719-20/B747-8; emphasis added)

Here Kant plainly conceives of an exhaustive division among *a priori* concepts: some are constructible and the others are not. But he also plainly conceives of the non-constructible pure concepts as the categories, or pure concepts of the understanding: those which contain 'nothing but the *synthesis* of possible intuitions, which are not given *a priori*'. This leaves only pure *sensible* concepts as candidates for constructibility.

That pure sensible concepts are 'constructible' is relevant to my suggestion that a concept of SPACE governs our capacity to intuit individual spaces. Pure sensible concepts comprise the concept of SPACE *and* concepts of spaces;[14] that is, pure sensible concepts comprise the aesthetic concept of SPACE as I have described it, as well as any and all pure spatial concepts, such as straight line or triangle. For Kant, what it means to 'construct' any such concept is 'to exhibit *a priori* the intuition corresponding to it' (A713/B741). Taking this together with the passage above suggests that the concept of constructibility comes to this: to possess a *constructible a priori* concept is to be able to exhibit some pure intuition that corresponds to that concept in an as yet unspecified way. But a concept such as triangle (which we can imagine to include a geometric procedure for constructing triangles) is insufficient on its own to explain our ability to construct a triangle in pure intuition. This

---

[12] These are, of course, distinctive features of Kantian concepts.

[13] Notice that on my suggested interpretation, the 'general concept of spaces in general' plays a role in gathering intuitions of individual spaces, but is insufficient to account for the construction of such.

[14] As well as the concept of TIME and concepts of times.

is because our ability to construct the concept triangle by exhibiting a triangular figure is not exhausted by our possession of the concept of triangle, but requires also our possession of the concept of SPACE. I am not suggesting that we *construct* SPACE in order to construct individual spaces, such as triangles; only individual (finite) spaces can be *constructed*, as Kant understands that term. Rather, we construct individual spaces, such as triangles, under constraints and conditions imposed by a concept of SPACE; a concept of SPACE necessarily underlies any potentially exhibitable intuition of an individual spatial region. It is this concept of SPACE, which provides a principle for the construction of individual spaces, that is the subject of the Metaphysical Exposition.[15]

Suppose Kant conceives the concept of SPACE as something like a principle governing the intuitions of individual spaces; suppose further that he conceives our concepts of individual spaces to be constructible in the sense described above; it follows that to fulfill its stated goal of exhibiting that which is given in the concept of SPACE, the Metaphysical Exposition ought to identify that aspect of our faculty of *a priori* sensibility that warrants and constrains the constructibility of individual spaces. It remains for us to examine the content of this concept of SPACE, that I allege to play a pre-constructive role in our geometrical cognition.[16]

2.

Kant takes us to possess a sensible capacity to represent space intuitively; exercise of this capacity to intuit particular spatial regions takes place in accordance with certain constraints and conditions. These constraints and conditions are, in my view, articulated in the course of the four arguments of the Metaphysical Exposition, which have commonly been understood to demonstrate both the *a priori* and intuitive nature of our representation of space. It is not my task here to provide an analysis or interpretation of these arguments; rather, I will extract from the arguments the claims Kant makes that illustrate the constraints and conditions on our capacity to represent space intuitively, and which serve thereby as a partial exposition of our concept of SPACE.[17]

In the course of his first argument for the intuitive nature of our representation of space, Kant writes:

[15] Thus, if we were to speak of the constructibility of the *concept* of SPACE we would mean to designate the constraints and conditions on the constructibility of spatial concepts, such as triangle or line, that is, the constraints and conditions on our capacity to exhibit a pure intuition of a triangle or line via construction.

[16] By this I mean that geometrical construction cannot proceed without that which is provided by an independent concept of SPACE. (Of course, geometrical construction also cannot proceed without a capacity for productive imagination. Investigation of the relation between imagination and geometrical construction is beyond the scope of this paper.)

[17] The exposition is partial because these claims are not sufficient on their own to describe the *a priority* of space, but only its intuitivity. Our purposes will be adequately served by considering only the two arguments used to demonstrate the intuitive nature of our representation of space: arguments 4 and 5 in the A-edition, and 3 and 4 in the B-edition.

> . . . one can only represent a single space, and if one speaks of many spaces, one understands by that only parts of one and the same unique space. And these parts cannot as it were precede the single all-encompassing space as its components (from which its composition would be possible), but rather are only thought **in it**. It is essentially single; the manifold in it, thus also the general concept of spaces in general, rests merely on limitations. (A25/B39; emphasis original)

In the course of his second argument for the intuitive nature of our representation of space, Kant writes:

> Space is represented as an infinite **given** magnitude[18]. . . but no concept, as such, can be thought as if it contained an infinite set of representations within itself. Nevertheless space is so thought (for all the parts of space, even to infinity, are simultaneous). (A25/B40)

In the A-edition, he adds:

> If there were not boundlessness in the progress of intuition, no concept of relations could bring with it a principle of their infinity. (A25)

On first reading, these remarks seem to imply that our representation of SPACE is a representation of a *single* (and unique) whole, which is *infinite*, or unbounded in extent. And on first reading, it is difficult to know what to make of this. It is impossible that such features be represented in a *concept*, for the very reasons Kant cites: concepts are not singular, nor can they contain infinitely many parts. Thus, space is represented in intuition. But this seems just as problematic: it seems equally impossible to *intuit* a single infinitely large object.[19] According to Kant's own theory of intuition, this would require that we be able to form an immediate (unmediated) representation of an infinite spatial magnitude, that we grasp its infinitude in a single 'glance', as it were.

But if SPACE is construed as a principle governing the construction of finite spaces in intuition, then the claim that SPACE is a representation of a single and infinite whole serves both to *warrant* and *constrain* such construction by describing the peculiar conditions to which our spatial sensibility conforms. That is, the claim that SPACE is a representation of a single and infinite whole is a claim about what can and cannot be constructed in spatial intuition. So, when we exercise our capacity to intuit spaces, for example by constructing a triangle, we find that we cannot intuit individual spatial regions[20] except as finite parts of a larger surrounding spatial

[18] In the A-edition, this phrase is rendered 'given infinite magnitude' (A25).

[19] And problematic not only for the reason discussed above, namely that to exposit the *concept* of SPACE is to identify it as an *intuition*.

[20] A two-dimensional spatial region is the area bounded by a closed plane figure (e.g., a triangle and its interior).

region. When I construct a triangle,[21] I am immediately cognizant of the region bounded by the triangle, but I am also cognizant of the region that surrounds and contains the triangle. Suppose I conceive of the region that surrounds and contains the triangle as a square, whose side is twice as long as the longest side of the triangle. Upon constructing such a square, I am immediately cognizant of the region bounded by the square, which includes the triangular region, but I am also cognizant of the region that surrounds and contains the square. And so on.[22] Such an exercise of my capacity to construct finite spaces in intuition is accompanied by simultaneous cognizance that the constructed spaces are parts embedded in a uniform whole; the indefinite iterability of this exercise testifies that the whole fails to be exhausted by the mereological sum of these parts, so that the parts are seen as embedded in an *infinite single* whole. Ultimately, then, Kant construes infinitude and singularity to be features of our capacity to represent spatially: we are both warranted and constrained to represent spatial regions as finite sub-spaces of an infinite single whole.

Kant's conception of the uniformity and connectedness of space is the result of his engagement with standard modern mathematical practices: early modern geometers, like Euclid himself, conceive each successive dimension of space to be limited or bounded by the previous. So, the sense in which points are conceived as parts of lines is as the *limits* or *boundaries* of line segments. Likewise, line segments are the limits or boundaries of surfaces; surfaces of volumes. Kant's notion that finite spatial regions (and their corresponding concepts, e.g., a triangular figure and the concept of triangle) are merely 'limitations' on a single whole determined by the concept of SPACE corresponds to this conception: he means to describe each part of space—each particular spatial region—as itself a limit or boundary of some (infinitely) larger space. That we are *warranted* to represent ever larger finite spatial regions as limiting parts of an *infinite* space, and that we are *constrained* to represent finite spatial regions as limiting parts of a *single* space captures the content of our *a priori* concept of SPACE.

More generally, by identifying these features of our concept of SPACE, the arguments of the Metaphysical Exposition serve to unearth the intuitive foundations of our 'happy and well grounded' mathematical practices (A713/B741). It is important to note that the exercise of our intuitive capacity that is described by Kant's claims in the Metaphysical Exposition is not *itself* warranted or constrained by the science of geometry. Rather, it is quite the reverse: the practice of geometry is warranted and constrained by the intuitive capacity described by Kant's claims in the Metaphysical Exposition. That is, our capacity to represent spatial regions in the way just described provides us with a necessary foundation for geometric investigation: exercise of this capacity reveals the most basic or primary components of our spatial representations,

[21] By, literally, drawing in thought three distinct straight lines, each of whose endpoints is identical to an endpoint of one of the other two lines. Notice that one does not need to know the definitions or axioms of geometry to perform such a construction.

[22] Notice that this exercise could just as easily be carried out in three dimensions, thus extending the topological picture: instead of plane figures, imagine the exercise performed on cubic or spheric regions, so that, for example, cubes are embedded in spheres, embedded in cubes.

codification of which provides us with the self-evident ‘axioms’ of a science of space. This makes some sense of Kant’s apparently odd claim that the concept of SPACE is itself ‘a principle from which insight into the possibility of other synthetic *a priori* cognitions can be gained’ (B40): the self-evident ‘axioms’ that make a science of space *possible* are gleaned from the exercise of our capacity for intuiting space.

So, we have a capacity to intuit finite spatial regions as limitations on an infinite single whole space. The intuitions of such finite spatial regions begin with the simplest, most ‘primary’, constructions. In drawing a finite line in thought, I immediately cognize the medium in which the line is constructed, i.e., the larger spatial region which warrants indefinite production of the line from either endpoint *and* the two planar regions that the (indefinitely extended) line serves to bound. In drawing a finite line in thought, I cognize the primitive features of the space that the science of geometry will ultimately describe in full; codification of these primitive features would serve to provide the starting points for that science. It is in this sense that the ‘axioms’ of geometry originate in such primitive constructions: the ‘axioms’ are the primitive facts about space made evident by the exercise of our capacity to perform primitive spatial constructions.

The ‘axioms’ of an *a priori* science of space are, then, best described as self-evident *starting points* for the deduction of geometric theorems and solution of geometric problems. In the case of Euclid’s geometry, these ‘axioms’ include the definitions, postulates and common notions that precede the first proposition of the first book of the *Elements*. In the eighteenth century, presentation of these ‘axioms’ varied widely: Euclid’s own definitions were often accompanied by exegeses and diagrammatic renderings; his five postulates were typically restricted to claims that warrant diagrammatic constructions; and his common notions were typically extended to include the non-constructive postulates as well as whatever other claims the particular editor took to be ‘self-evident’.[23] Discussion of the details of these variations is beyond the scope of this paper; I wish to emphasize here that the ‘axioms’ of Euclid’s geometry include descriptive claims *and* constructive warrants and constraints that together codify our ability to construct and exhibit finite spatial regions in intuition.

The axioms of geometry codify the ‘principles’ governing our capacity to intuit regions of space, and thereby ground a science of space, by setting out the most general ‘rules’ for performing spatial constructions.[24] But these axioms cannot be expressed prior to our exercise of a primitive or original capacity to intuit finite spatial regions as parts of a single infinite whole given space. So, Kant uses the Metaphysical Exposition, at least in part, to describe the pure spatial intuition that underlies any and all geometric procedures, but he does not use properly geometric procedures to describe that intuition. While cognition of the ‘axioms’ of geometry

[23] For example, that a straight line cannot first come nearer to a straight line, and then go farther from it without cutting it. For details concerning the eighteenth-century reception of Euclid’s *Elements*, see Shabel (2002).

[24] Or, more accurately, by setting out the most general rules for constructing diagrammatic representations of spatial regions.

depends, in some sense, on our having a capacity for pure spatial intuition, that capacity cannot itself be described as a capacity for *geometric* reasoning. So, our capacity for pure spatial intuition, described in the Metaphysical Exposition, is pre-geometric in the sense that it is independent of and presupposed by Euclidean reasoning.

This last claim must be reconciled with Kant's sole mention of geometrical principles in the Metaphysical Exposition. After concluding the first argument for the intuitivity of space,[25] Kant writes:

> Thus also all geometrical principles, e.g., that in a triangle two sides together are always greater than the third, are never derived from general concepts of line and triangle, but rather are derived from intuition and indeed derived *a priori* with apodictic certainty. (A25/B39)

I take this to be a statement of what follows *from* the claim just established, namely, that space is an intuition, and not to play any role in the establishment of that claim. The intuitivity of space has just been demonstrated on the basis of the earlier claim that the parts of space 'cannot as it were precede the single all-encompassing space as its components'; this is not an argument that rests on any principle of geometry, but rather on what is self-evidently contained in our pure cognition of space. Kant clarifies the intuitivity result with the remark, immediately preceding the cited passage, that

> It [space] is essentially single; the manifold in it, thus also the general concept of spaces in general, rests merely on limitations. From this it follows that in respect to it an *a priori* intuition (which is not empirical) grounds all concepts of it. (A25/B39)

Kant then proceeds to remark that it follows from the intuitivity of space—and, in particular, from the relations between the concept of SPACE, the 'general concept of spaces in general', and pure spatial intuition—that all *geometrical* principles are themselves derived from pure spatial intuition. While the concept of SPACE governs our intuitive capacity for representing finite spatial regions as parts of a single infinite whole, the intuitions that result from an exercise of that capacity themselves prompt the derivation of geometric principles. Kant's argument is that these principles cannot be seen as deriving from any 'general concept of spaces in general' (geometric concepts such as line and triangle) but must be seen as deriving from our intuition of the space in which the concepts of line and triangle are themselves constructed.[26] So, the content of our geometric principles, like their constituent geometric concepts,

[25] Argument 3 in the B-edition.

[26] I am here using Kant's own terminology, according to which a *concept* is constructed via the exhibition of an intuition corresponding to that concept. So to construct the concept line is to draw a line in thought, thereby exhibiting it in intuition; to construct the concept triangle is to draw a triangle in thought, thereby exhibiting it in intuition.

must be exhibited in the medium provided by our pure spatial intuitions. But, then, geometric principles cannot play a role in describing that medium; only an exercise and inspection of our capacity for intuiting in that medium can play such a role.

In the B-Introduction, Kant identifies mathematics as providing a 'splendid example of how far we can go with *a priori* cognition':

> Now [mathematics] is occupied, to be sure, with objects and cognitions only so far as these can be exhibited in intuitions. This circumstance, however, is easily overlooked, since the intuition in question can itself be given *a priori*, and thus can hardly be distinguished from a mere pure concept. (A4/B8)

The Euclidean geometer, in particular, might be 'occupied with' a spatial region that can be exhibited in an intuition via construction of a two-dimensional geometric figure.[27] Kant claims here that because such an intuition is 'given *a priori*' it is difficult to see how it is to be distinguished from a 'mere pure concept' that is also 'given *a priori*'.[28] Kant here directs our attention to a distinction easily overlooked, between pure spatial intuitions and the concept of SPACE that underlies such intuitions; attention to this distinction helps to clarify both Kant's goals in the Metaphysical Exposition of the Concept of Space, as well as the role that that Exposition plays in founding mathematical practice. While the pure concept of SPACE is a given *principle* of *a priori* sensibility, the pure spatial intuitions governed thereby are better described as '*exhibited*' rather than '*given a priori*'. That is, exercise of our capacity for spatial intuition (which is warranted and constrained by the pure concept of SPACE) enables the *exhibition* of pure spatial intuitions, without which the mathematician would be left wholly unoccupied.

Acknowledgements

I am grateful to Emily Carson and Daniel Sutherland for penetrating comments and suggestions on earlier drafts, and to William Taschek for helpful discussions.

References

Allison, H. (1983). *The Kant–Eberhard controversy*. Baltimore: Johns Hopkins University Press.
Friedman, M. (1992). *Kant and the exact sciences*. Cambridge: Harvard University Press.
Friedman, M. (2000). Geometry, construction and intuition in Kant and his successors. In G. Sher, & R. Tieszen (Eds.), *Between logic and intuition: Essays in honor of Charles Parsons* (pp. 186–218). Cambridge: Cambridge University Press.

[27] Or, more accurately, with a spatial region that can be exhibited in an intuition via construction of a diagrammatic representation of a geometric figure.

[28] Thus, I read 'can hardly be distinguished from' as 'is difficult to distinguish from' ('mithin von einem bloßen reinen Begriff kaum unterschieden wird').

Kant, I. (1998). *Critique of pure reason* (P. Guyer, & A. Wood, Trans.). Cambridge: Cambridge University Press.

Shabel, L. (1998). Kant on the ‘symbolic construction’ of mathematical concepts. *Studies in History and Philosophy of Science, 29A,* 589–621.

Shabel, L. (2002). *Mathematics in Kant’s critical philosophy: Reflections on mathematical practice*. New York: Routledge.

Woolhouse, R. S. (Ed.). (1988). *Metaphysics and philosophy of science in the seventeenth and eighteenth centuries: Essays in honour of Gerd Buchdahl*. Dordrecht: Kluwer Academic Press.