Kant's "Argument from Geometry"

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IN INTERPRETING THE IMPORTANT section of the *Critique of Pure Reason* entitled "Transcendental Exposition of the Concept of Space," it has long been standard to suppose that Kant offers a transcendental argument in support of his claim that we have a pure intuition of space. This argument has come to be known as the "argument from geometry" since its conclusion is meant to follow from an account of the possibility of our synthetic *a priori* cognition of the principles of Euclidean geometry. Thus, Kant has been characterized as having attempted to deduce a theory of space as pure intuition from an assumption about mathematical cognition.

Insofar as Kant's theory of space is typically taken to be a central tenet of his transcendental idealism, the "argument from geometry" has come to bear a heavy burden. Indeed, many if not most commentators agree that Kant's transcendental idealism is unsupported by the "argument from geometry," arguing that history has shown it to have buckled under the weight of developments in mathematics and mathematical rigor unforeseen to Kant. Even commentators who disagree about the ultimate status of Kant's idealist conclusions agree about the inability of the "argument from geometry" to sustain them. So, for example, Henry Allison holds that the arguments for transcendental idealism succeed independently of the failed "argument from geometry" while Paul Guyer indicts the former due to what he argues is their direct dependence on the latter.[1]

It is important to notice, however, that the standard interpretation of the "argument from geometry" takes Kant to be arguing in the "analytic" or "regressive" style that he assumes in his *Prolegomena.* Such an argument begins with some body of knowledge already known to have a certain character, such as mathematics, in order to "ascend to the sources, which are not yet known, and whose discovery not only will explain what is known already, but will also exhibit an area with many cognitions that all arise from these same sources."[2] Thus, on the standard inter-

[1] Paul Guyer, *Kant and the Claims of Knowledge* (Cambridge: Cambridge University Press, 1987), 367; Henry Allison, *Kant's Transcendental Idealism* (New Haven: Yale University Press, 1983), 99.

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pretation of the "argument from geometry," Kant analyzes our synthetic *a priori* knowledge of Euclidean geometry in order to discover what is not yet known: that we have a pure intuition of space. But this interpretation is in direct conflict with Kant's own stated claim to be providing "synthetic" or "progressive" arguments in the *Critique*, arguments that "develop cognition out of its original seeds without relying on any fact whatever."[3]

Though the ultimate defensibility of Kant's doctrine of transcendental idealism is not my immediate concern in what follows, I propose nevertheless to defend an alternative reading of the "argument from geometry," an argument that I construe as synthetic and so not transcendental in the standard sense. In reinterpreting the "argument from geometry," I thereby reassess the role of geometric cognition in the arguments of the "Aesthetic," showing that Kant's philosophy of geometry builds a philosophical bridge from his theory of space to his doctrine of transcendental idealism. The "argument from geometry" thereby exemplifies a synthetic argument that reasons progressively *from* a theory of space as pure intuition, offered in the earlier "Metaphysical Exposition of the Concept of Space," to a theory of geometry and, ultimately, to transcendental idealism. Kant's own metaphor will thus guide us in showing that our pure intuition of space provides the seeds for our cognition of the first principles of geometry.

The "argument from geometry" does not analyze geometric cognition in order to establish that we have a pure intuition of space. Rather, the "argument from geometry" establishes that geometric cognition itself develops out of a pure intuition of space. The difference is subtle, but important: on the standard reading, our actual knowledge of geometry is traced to its source—namely, a pure intuition of space—in order to show that we must, therefore, have such a pure intuition. On my reading, our pure intuition of space is offered as both the actual source of our cognition of the first principles of geometry and the means for the production of further cognition based thereon.[4] The "argument from geometry" so understood reveals a coherent and compelling philosophy of geometry that must be taken itself as a primary component of Kant's project in the first *Critique*. More importantly, perhaps, the "argument from geometry" so understood serves to illustrate and clarify Kant's notion of a synthetic deduction—a notion that has great interpretive influence over many subsequent arguments in the *Critique*.

In the first section, I will outline what I take to be the argument structure of the "Aesthetic," for the purpose of providing a context for a discussion of the "argument from geometry." In the second section, I will analyze and interpret the arguments in the "Transcendental Exposition of the Concept of Space" to show that, contrary to the received view, the so-called "argument from geometry" iden-

[2] Immanuel Kant, *Prolegomena to Any Future Metaphysics*, Gary Hatfield, ed. and trans. (Cambridge: Cambridge University Press, 1997), 26 [Ak 4:275]. Subsequent references to *Prolegomena* cite volume and page number of the Akademie edition *Kant gesammelte Schriften,* (Berlin: G. Relmer, 1910–).

[3] Ibid., 25 [Ak 4:274].

[4] In this paper, I aim to show that Kant means to establish *that* a particular relation holds between our pure intuition of space and our cognition of geometry, as described above. An account of the details of this relation, that is, an account of exactly *how* our pure intuition of space affords us our cognition of the first principles of geometry, will require further work on the role of the faculty of productive imagination in constructing the objects of geometry.

tifies the essential role of pure spatial intuition in geometric cognition. Finally, in the third section, I will discuss the early modern reception of a theorem of Euclid's geometry, in order to illustrate the origin of Kant's understanding of the role of pure intuition in geometry.

My overarching aim is to reinterpret Kant's "argument from geometry," which I take to show that Euclid's geometry as Kant understood it is *grounded* in a pure intuition of space. My interpretation requires that I distinguish three Kantian claims that are typically conflated, any one of which understood in isolation might plausibly be taken to illustrate Kant's doctrine of transcendental idealism, but none of which can plausibly be construed as the conclusion of the "argument from geometry":

[1] Space is a pure intuition;
[2] Space is a pure form of sensible intuition;
[3] Space is only as described in [1] and [2].

I take [1] to be equivalent to the claim that our representation of space is absolutely *a priori* and non-conceptual, or that space is represented to us as a pure intuition. I take [2] to be equivalent to the claim that the representation described in [1] provides us with a partial structure for cognizing empirical objects. I take [3] to express a further and independent claim that space is itself nothing over and above the structure described in [2], i.e. that space is transcendentally ideal.[5] What I hope to show is that, rather than serving as an argument for any or all of the above claims, Kant's so-called "argument from geometry" establishes the relation between our representation of space (as expressed in [1]) and our cognition of geometry, thus allowing geometry, the mathematical science of space, to play a role in subsequent arguments for [2] and [3].

Accordingly, Kant gives arguments for [1] that are independent of considerations about geometry;[6] he then proceeds to show (in the passages that constitute the so-called "argument from geometry"[7]) that the representation of space as described in [1] provides the foundation for pure geometric cognition. In the subsequent passage,[8] he claims that the pure geometric cognition that is so founded is *empirically applicable* only if [2] holds. And in subsequent sections[9] he provides direct support for [3], thus completing a series of arguments that together form a defense of the doctrine of transcendental idealism. On my interpretation, the "argument from geometry," though providing only one step on the path to Kant's arguments for the doctrine of transcendental idealism, nevertheless plays the crucial philosophical role of connecting Kant's metaphysical theory of space as pure intuition with his mathematical theory of pure geometry.

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[5] Notice that [1] and [2] do not include commitments to the nature of space itself, that is to whether or not space as we represent it (described by [1] and [2]) is space as it is independently of that representation.

[6] In the "Metaphysical Exposition of the Concept of Space."

[7] In the second paragraph of the "Transcendental Exposition of the Concept of Space."

[8] In the third paragraph of the "Transcendental Exposition of the Concept of Space."

[9] In the "Conclusions from the Above Concepts."

1. THE ARGUMENT STRUCTURE OF THE "AESTHETIC"

Recall that Kant's aim in the "Transcendental Aesthetic" is to provide a "science of all principles of *a priori* sensibility" (A21/B36).[10] By "*a priori* sensibility" Kant means to isolate a capacity to represent certain features of the objects of our possible experience, namely those features that are intuitively given but non-empirical. A "science" of such a capacity would thus provide—at a minimum—a codification of the first principles of cognition that are both *a priori* and intuitive.[11] Kant asserts from the start that such *a priori* intuition is necessarily separable from empirical sensation and so can be investigated independently therefrom: "since that within which the sensations can alone be ordered and placed in a certain form cannot itself be in turn sensation, the matter of all appearance is only given to us *a posteriori*, but its form must all lie ready for it in the mind *a priori*, and can therefore be considered separately from all sensation" (A20/B34).

Thus, Kant begins the "Aesthetic" by authorizing an investigation of pure intuition (or, as he also calls it, "the pure form of sensibility"),[12] an investigation that immediately commences with reflection on the experience of representing a material body:

> . . . if I separate from the representation of a body that which the understanding thinks about it, such as substance, force, divisibility, etc., as well as that which belongs to sensation, such as impenetrability, hardness, color, etc., something from this empirical intuition is still left for me, namely extension and form. These belong to the pure intuition, which occurs *a priori*, even without an actual object of the senses or sensation, as a mere form of sensibility in the mind. (A21/B35)

So, the science that Kant is pursuing in the "Aesthetic" will account for the pure intuition of "extension and form" that cognitively precedes and enables our empirical intuition of material bodies. Notoriously, Kant identifies space and time as the representations afforded us by pure intuition, representations that are strictly independent of any representations we may subsequently acquire of spatial or temporal objects. Moreover, he takes space and time to be the "principles" of pure cognition and proceeds to their "assessment."

The assessment begins with the "Metaphysical Exposition of the Concept of Space,"[13] wherein Kant provides two arguments showing that space is a "pure" or absolutely *a priori* representation: when we cognize space "nothing empirical is intermixed" (B3) and, moreover, our representation of space is a necessary condition on our representation of outer objects. He follows with two arguments showing that space is an intuition: our cognition of space is non-conceptual cog-

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[10] All citations to Kant's *Critique of Pure Reason* will be made using the usual A/B convention. Translations are from Immanuel Kant, *Critique of Pure Reason*, Paul Guyer and Allen Wood, eds. (Cambridge: Cambridge University Press, 1998).

[11] For a brief discussion of the meaning of 'science' in Kant's time, see Gary Hatfield's "Notes on terminology" in his introduction to the Cambridge edition of the *Prolegomena* (Kant 1997, xxiii).

[12] When speaking of the capacity, or faculty, itself, Kant sometimes conflates the terminology, thus suggesting that our capacity of pure intuition *is* the "pure form" of our faculty of sensibility. This conflation is clarified somewhat by separating the more specific claims about our representation of space, as I have done above.

[13] There are, of course, parallel arguments concerning the concept of time, which I will not address here.

nition of an essentially single and infinite given magnitude. Taken together, these arguments are meant to establish what I have identified as claim [1], above: Space is a pure intuition.

The literature analyzing and evaluating these arguments is voluminous. Several recent analyses have significantly deepened our understanding of the merits of Kant's position here; I follow, in particular, Daniel Warren[14] and Emily Carson[15] in considering these arguments to be viable defenses of the *a priori* and intuitive nature of our representation of space. For our purposes, what is of interest is what Kant takes himself to have accomplished in offering these arguments at this stage. So, before continuing to the "Transcendental Exposition of the Concept of Space," I will back up briefly to identify the larger goal which Kant takes these four arguments to satisfy.

In order to understand the nature of space, Kant demands an exposition of its concept (which he meets with the four arguments mentioned above):

> I understand by exposition [*expositio*] the distinct (even if not complete) representation of that which belongs to a concept; but the exposition is metaphysical when it contains that which exhibits the concept as given *a priori*. (B38)

Though his burden here is not obvious, he seems to regard the characterization of our representation of space offered by the four arguments just cited as at least partially descriptive of space itself. By expositing our constraint to represent space as, for instance, infinite we thereby "exhibit" the limitlessness of spatial extension. Likewise, by expositing our constraint to represent space as single and unique, we thereby "exhibit" the simultaneity of the parts of space. It seems then that the "Metaphysical Exposition" seeks to identify the primitive features of spatiality to which we have *a priori* cognitive access by "exhibiting" those very features as given in our representation of space.[16]

Continuing now to the crucial section, we find that Kant distinguishes a *transcendental* exposition from a *metaphysical* exposition in the following way:

> I understand by a transcendental exposition the explanation of a concept as a principle from which insight into the possibility of other synthetic *a priori* cognitions can be gained. For this aim it is required 1) that such cognitions actually flow from the given concept, and 2) that these cognitions are only possible under the presupposition of a given way of explaining the concept. (B40)

Immediately following this statement is the so-called "argument from geometry," which I will discuss in detail in the following section. Here let me simply assess the motivation for introducing that argument.[17]

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[14] Daniel Warren, "Kant and the Apriority of Space," *Philosophical Review* 107 (1998): 179–224.

[15] Emily Carson, "Kant on Intuition in Geometry," *Canadian Journal of Philosophy* 27 (1997): 489–512.

[16] But it remains puzzling that Kant should think that spatial features might be exhibited independently of spatial objects. That is, we might question our ability to represent spatial relations that "only attach to the form of intuition alone" (A23/B38): what is it like to intuit relations without any corresponding intuition of the relata? This is a question that I hope to answer on Kant's behalf below.

Incidentally, Kant's claim that we "exhibit" the concept of space is illuminated by his discussion in the "Discipline of Pure Reason in its Dogmatic Use," where he distinguishes the pure *sensible* concepts of space and time from the pure concepts of the understanding by claiming that the former, unlike the latter, are *a priori* constructible since they "already contain a pure intuition." (A719/B747) A discussion of these passages is beyond the scope of this paper.

[17] Recall that the A- and B-editions of the *Critique* differ in their placement of the "argument from geometry": in the former, Kant includes an earlier version as a fifth argument in the "Metaphysical

According to the above passage, a transcendental exposition of the concept of space aims to explain the sense in which our concept of space acts as a grounding *principle* for our acquisition of other synthetic *a priori* cognitions; as we will learn from the subsequent passage, these cognitions include geometry. Recall that in the preceding sections Kant takes himself to have shown that "the original representation of space" is an *a priori* or pure intuition. Since the concept of space has previously been shown to include certain features accessible to us via pure intuition, Kant here hopes to identify how those features and our cognition thereof can serve as the basis for further *a priori* cognition. That is, Kant takes the features of space isolated in the "Metaphysical Exposition" as a starting point and asks whether they can account for or ground any other cognition. Put yet another way, Kant asks whether pure intuition of the primitive features of space—infinitude and singularity—can be described as making possible any other *a priori* cognition.[18] Kant answers this question with the "argument from geometry," the object of our discussion below.

Finally, before proceeding to a similar pattern of argumentation with respect to the concept of time, Kant provides several further arguments for the claims that space is a pure form of sensible intuition, and nothing more, (identified above as [2] and [3]), thus providing his first clear articulation of the doctrine of transcendental idealism. In these sections, Kant shifts his attention from our pure intuition of space and the foundations of *pure* geometric cognition to our empirical intuition of spatial objects and the *applicability* of geometry, concluding that space can be "nothing other than merely the form of all appearances of outer sense, i.e., the subjective condition of sensibility" (A26/B42). Despite their textual independence from the passages that constitute the so-called "argument from geometry," many commentators take the success of these idealist conclusions regarding the nature of space to stand or fall with that argument. It is to that argument that we now turn.

2. THE "TRANSCENDENTAL EXPOSITION OF THE CONCEPT OF SPACE"

Kant's goal in this section is to provide a second "exposition" of the concept of space. In particular, he is here interested in the sense in which our cognition of space, constrained in the ways demonstrated by the "Metaphysical Exposition,"

Exposition of the Concept of Space," whereas in the latter he moves it to the new "Transcendental Exposition." This relocation is evidence of what I will try to show later, that the "argument from geometry" is not meant to show that space is a pure intuition. Realizing that the argument was misplaced in the A-edition, Kant reformulated and moved it to a new section in the B-edition, thus altering its role in the "Aesthetic." His stated goals for this new section are thus an important clue to the role of the "argument from geometry" on his revised view. I will briefly return to the A-edition argument in section 2 below, in order to provide further textual support for my interpretation.

[18] Distinct from the interpretive tradition mentioned above, according to which the "argument from geometry" plays a role in establishing *that* we have a pure intuition of space, there have also developed two standard approaches to understanding Kant's view of *how* intuition functions in mathematical cognition. The "logical" approach, identified primarily with Michael Friedman, and the "phenomenological" approach, identified primarily with Charles Parsons, differ in the emphasis that each places on *singularity* and *immediacy* as criteria of intuitive representations. It seems to me that Kant takes both singularity and immediacy to be primitive features of our representation of space, features that play inter-related roles in our cognition of the spatial objects of geometry. The singularity of our representation of space amounts to our inability to represent the uniqueness of space via a combina-

can provide fundamental insights into any other synthetic *a priori* cognition that we come to acquire. He requires that this insight meet two related demands: first, the cognitions that are explained by exposition of the given concept must "actually flow from" that concept. In the present context, this means that whatever cognitions are explained by an exposition of the concept of space are so explained because they "flow from" the concept of space. We are thus invited to reflect on the concept of space in order to determine whether and which synthetic *a priori* cognitions result. Second, whatever cognitions do result from such reflection "are only possible under the presupposition of a given way of explaining the concept." Again, in the present context, Kant here demands that a particular way of understanding the original concept of space must be a necessary condition on our acquiring the new cognitions. That is, the possibility that new synthetic *a priori* cognitions "flow from" the concept of space depends on our understanding the concept of space in a particular way. That Kant places these demands on his own exposition highlights the distinction he draws between a "metaphysical" exposition of a concept, which isolates that which is given *a priori*, and a "transcendental" exposition, which isolates what we cognize on the basis of that which is given *a priori*. Both such expositions describe *a priori* cognition of space: the first describes the unique features of our passive capacity for spatial intuition, while the second describes the result of reflection on and employment of that very capacity.

Having introduced his goal and its attendant conditions, Kant proceeds to the passage that includes the "argument from geometry." The passage includes two stages of argumentation corresponding to the two conditions on his exposition; I will treat these stages separately beginning with the first, which involves direct reference to geometric cognition:

> Geometry is a science that determines the properties of space synthetically and yet *a priori*. What then must the representation of space be for such a cognition of it to be possible? It must originally be intuition; for from a mere concept no propositions can be drawn that go beyond the concept, which, however, happens in geometry (Introduction V). But this intuition must be encountered in us *a priori*, i.e., prior to all perception of an object, thus it must be pure, not empirical intuition. For geometrical propositions are all apodictic, i.e., combined with consciousness of their necessity, e.g., space has only three dimensions; but such propositions cannot be empirical or judgments of experience, nor inferred from them (Introduction II). (B41)

An argument that has proved virtually indefensible is commonly extracted from this passage:

1. We have synthetic *a priori* cognition of Euclidean geometry
    Or: Euclidean geometry is necessarily true
2. Such cognition is possible only if space is a pure intuition
    Or: pure intuition of space is a necessary condition of our synthetic *a priori* cognition of geometry

Therefore, space is a pure intuition.[19]

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tion of unique parts; the immediacy of our representation of space amounts to our inability to represent the parts of space except as simultaneous with the whole infinite space of which they are parts. These features of our representation of space each play an indispensable role in grounding our systematic cognition of mathematics; I will investigate this role in more detail elsewhere.

[19] A paradigm such extraction is offered by Bertrand Russell in his essay "Kant's Theory of Space" (Bertrand Russell, *The Principles of Mathematics* [New York: Norton, 1937], 456). Though he admits

This rendering of Kant's argument suggests that he meant to show that our cognition of geometry affords us our cognition of space in the following sense: reflection on the status of our geometric cognition reveals a necessary feature of our representation of space, namely that it is a pure intuition. On this reading, of course, Kant draws a conclusion about the nature of our representation of space that depends directly on substantive assumptions about the nature of mathematical reasoning, assumptions that contemporary readers are reluctant to grant.

This is not, however, how Kant meant to argue in the foregoing passage. First, as we have noted, Kant's goals in the current section differ from his goals in the preceding "Metaphysical Exposition," where he takes himself *already to have shown that space is a pure intuition.* We should expect that his arguments in the separate sections—which have distinct aims—would not proceed toward the same conclusion.[20] Moreover, inspection of those aims reveals that Kant is not reasoning *on the basis of* geometric cognition, as the above reconstruction suggests, but rather on the basis of that which has already been shown to be given *a priori* in our concept of space. That is, in the "Transcendental Exposition" Kant is reasoning synthetically *from* the pure intuition of space exhibited in the "Metaphysical Exposition" *to* the possibility of synthetic *a priori* geometric cognition.

On my view, then, Kant's "argument from geometry" shows that our cognition of space affords us our cognition of geometry—roughly the reverse of what has long been alleged. That is, Kant's reasoning is premised on the conclusion drawn in the preceding section—namely, that space is a pure intuition—and directed at an explanation of how that conclusion explains another body of cognition, namely geometry. Recalling the aim specific to this section, Kant asks whether the pure intuition of space does (and must) serve as a principle for gaining insight into the possibility of other synthetic *a priori* cognition. Having shown that space is a pure intuition, it is as if Kant asks: what are we representing when we represent space? What knowledge does our representation of space afford us, and how?[21]

I interpret his answer as follows: first, Kant observes that we have synthetic *a priori* cognition of geometry, the science of space. His assumption that geometry is the science of space is uncontroversial: Kant and his contemporaries use the term 'mathematics' to designate the science of magnitude, or quantity, and 'geometry' to designate the sub-science of spatial magnitude.[22] The *a priority* of these

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that this argument is especially evident in the *Prolegomena*, he claims nevertheless to assess Kant's critical theory of space as it is expressed in both the *Prolegomena* and the *Critique*; he takes Kant's argument to be the same in both texts. See also P.F. Strawson, *The Bounds of Sense* (London: Routledge, 1966), 57 and C.D. Broad, *Kant: An Introduction* (Cambridge: Cambridge University Press, 1978), 45ff. There are many others.

[20] Henry Allison suggests that the arguments given in the "Transcendental Exposition" merely buttress arguments given in the "Metaphysical Exposition" (Allison 1983, 99). But, this does not account for Kant's having made very clear that the sections have different aims.

[21] Remember that at this stage of the argument, it is still an open question whether our representation of space is likewise a representation of features of things-in-themselves. Transcendental idealism has not yet been introduced or defended; so it is as yet unclear what Kant thinks a representation of space actually captures.

[22] For historical background to the status of elementary mathematical disciplines in the early modern period, see my "Kant on the 'symbolic construction' of mathematical concepts," *Studies in History and Philosophy of Science* 29A (1998): 589–621 and *Mathematics in Kant's Critical Philosophy: Reflections on Mathematical Practice* (New York: Routledge, 2003), especially part 2.

disciplines is not in dispute: during the early modern period Euclid's geometry is universally considered a paradigm body of *a priori* knowledge. The syntheticity of geometry is defended on the basis of examples from mathematical practice in many familiar passages elsewhere in the *Critique*.[23] Remembering the background claim that space is a pure intuition, Kant proceeds to ask: how does our representation of space manage to afford us those cognitions that are the unique domain of the science of geometry? His question is not *whether* it does so, but *how*.

His response includes recognizing that to have cognition of the properties of space, one must begin with a cognition of space itself; that is, the geometer must be able to cognize, or mentally represent, the object of the science of geometry in order to cognize, or mentally represent, the properties of that object. Because the original representation of the object of geometry, namely space, is *a priori* (as has been established in the "Metaphysical Exposition"), the cognition of its properties are likewise *a priori*. And because the original representation of the object of geometry, namely space, is an intuition (as has also been established in the "Metaphysical Exposition") the cognition of its properties are synthetic. So, the synthetic *a priori* character of the geometer's cognition of the properties of space is due to an ability to represent the object of geometry—space itself—in pure intuition.

Here Kant does not advance an argument for the ideality of space, nor even an argument for the specific claim that we represent space in pure intuition, as traditional interpretations allege. Rather, having already shown that we represent space in pure intuition, Kant here describes the sense in which this representation relates to our cognition of geometry, the science of space, and our synthetic *a priori* cognition thereof. In particular, he advances a theory according to which geometrical cognition is founded on and generated from the already established pure intuition of space. It follows that the first principles of geometry, as well as the theorems deducible therefrom, all originate in a pure intuition of space. Support for this theory results from reflection on the contradictions that would result from supposing that the geometer's original representation of space were (*contrary to what has already been established*) neither intuitive nor *a priori*. If the original representation of space were a concept,[24] then the geometer's demonstrations would not be synthetic since the properties of space would follow from mere analysis of the concept of space. But this contradicts mathematical practice, which finds the geometer employing constructive procedures to demonstrate synthetic propositions.[25] If the original representation of space were empirical, then the geometer's propositions would fail to be apodictic. But this likewise contradicts mathematical practice, which takes pure mathematical knowledge to be a paradigm of certainty.

What Kant means to establish here is the relation between two cognitively significant abilities, namely, our pure intuition of space and our cognition of geom-

[23] See, for example, passages at (B14–B17) and (A713/B741).

[24] In particular, a concept that does not "already contain a pure intuition" and so a concept that is not uniquely constructible. See n. 16, above.

[25] Kant describes both the geometric *demonstrations* and geometric *principles or propositions* as "synthetic." The geometer's reasoning in the course of demonstration is synthetic because it depends on inferences made on the basis of constructed figures rather than mere analysis of concepts. The original principle or proved proposition is synthetic because it expresses a relation among concepts that are not analytically contained one in the other. Obviously, these two uses of "synthetic" are consistent.

etry. In particular, he establishes that our cognition of geometry *depends* on our having a pure intuition of space; that is, our pure intuition of space provides a cognitive foundation for our synthetic *a priori* cognition of geometry. This is to deny that the "argument from geometry" derives the fact of our pure intuition of space from the possibility of our cognition of geometry. That our cognition of geometry depends in some epistemically fundamental sense on our having a pure intuition of space is thus what it means to say that the cognitions of geometry "actually flow from" that which is given *a priori* in our concept of space.[26] Recalling the aim of the "Transcendental Exposition," the concept of space has thus been shown to meet the first condition on a principle that affords insight into the possibility of geometric cognition.

Further textual and philosophical support for my reading of Kant's "argument from geometry" is to be found in the first (A) edition version of this same material. In the A-edition of the *Critique*, Kant makes no distinction between a "Metaphysical" and "Transcendental Exposition" choosing simply to present five arguments "On Space." Four of these five arguments correspond to the four arguments in the second (B) edition's "Metaphysical Exposition." The fifth (which is presented after the two *a priority* arguments and before the two intuitivity arguments) can be assimilated to the B-edition's "Transcendental Exposition." Thus, in seeking to understand the argument of the "Transcendental Exposition," it is instructive to look to its ancestor:

> The apodictic certainty of all geometrical principles and the possibility of their *a priori* construction are grounded in this *a priori* necessity. For if this representation of space were a concept acquired *a posteriori*, which was drawn out of general outer experience, the first principles of mathematical determination would be nothing but perceptions. They would therefore have all the contingency of perception, and it would not even be necessary that only one straight line lie between two points, but experience would always merely teach that. What is borrowed from experience always has only comparative universality, namely through induction. One would therefore only be able to say that as far as has been observed to date, no space has been found that has more than three dimensions. (A24)

Kant's habit in all of these arguments is to begin with a statement of his conclusion. Thus, he wants here to show that the *a priori* character of space (demonstrated in the preceding two A-edition arguments) *grounds* the certainty of geometrical principles as well as the *a priori* constructions on which cognition of these principles depends. This conclusion, if extended to depend on both the *a priori* and the intuitive character of space, is what I claim to be the conclusion of the first part of the B-edition's "Transcendental Exposition." His argument here, in the A-edition, begins by supposing that space were not *a priori*, as he has just shown that it is, but rather *a posteriori*. On this assumption, it would follow that the axioms of geometry would be empirically perceptible. In particular, the geometer's claim that only one straight line lies between two points would be the contingent result of empirical observation, and thus subject to revision in the face of conflicting experience. But then, of course, the axioms would not be *axioms*: they would not be the necessary and universal truths that mathematical practice demands.

[26] I realize that we still need an account of how this works, of what it is like for the demonstration of a geometric theorem to "flow from" a pure intuition of space. A partial account will be given in section 3 below. I expect to provide a full account elsewhere.

Kant's argument here is not a *reductio* meant to establish the *a priority* of space;[27] indeed, the preceding two arguments have already established that. Instead, by showing that the *a posteriority* of space would *fail* to account for the axioms of geometry, he shows that the *a priority* of space *succeeds*. That is, he shows that an *a priori* concept of space is the source of the success of the geometrical method of reasoning. Since we know further that the geometrical method depends on judgements relating constructible pure concepts, it follows that: "The apodictic certainty of all geometrical principles and the possibility of their *a priori* construction are grounded in this *a priori* necessity."[28]

Returning now to the "Transcendental Exposition" as it was added in the B-Edition, Kant proceeds to the second condition on such an exposition, in order to show that the cognitions of geometry "are only possible under the presupposition of a given way of explaining the concept" of space. Kant makes this second stage of the argument in the paragraph that follows:[29]

> Now how can an outer intuition inhabit the mind that precedes the objects themselves, and in which the concept of the latter can be determined *a priori*? Obviously not otherwise than insofar as it has its seat merely in the subject, as its formal constitution for being affected by objects and thereby acquiring immediate representation, i.e., intuition, of them, thus only as the form of outer sense in general. (B41)

Here Kant points to what he takes to be a sort of paradox made evident by reflection on our geometric cognition. It has just been established that the principles of geometry describe and codify my *a priori* cognition of space and spatial relations. But the properties of space so described are instantiated by the sensible features of spatial, i.e. *outer*, objects. So, the spatial orientations among pure geometric figures correspond naturally to the spatial orientations among empirical objects.[30]

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[27] That is, the argument must *not* be interpreted as having the following structure: Suppose space is *a posteriori*; then geometrical principles would not be apodictic. Therefore, space is *a priori*.

[28] Despite the difference in its argumentative strategy, there is further support for this reading of the content of Kant's view in his *Prolegomena*, where he states that it is in the nature of mathematics that "it must be grounded in some *pure intuition* or other, in which it can present, or, as one calls it, *construct* all of its concepts *in concreto* yet *a priori*" [4:281]. In the case of geometry, a pure intuition of space *grounds* geometric reasoning by providing the medium for constructing geometric concepts [4:283]. Later, Kant unambiguously claims that the representation of space is "a representation that serves *a priori* . . . as foundation for the geometer" [4:283] and further that the propositions of the geometer are "extracted" from pure spatial intuition. In fact, I think that his position regarding the *role* of pure spatial intuition in mathematical reasoning is the same across both works: pure spatial intuition grounds mathematical reasoning. But the argument that he uses in the *Prolegomena*, which moves *from* the "uncontested" science of mathematics *to* its cognitive sources cannot be assimilated to the argument in the *Critique*. The former argument derives the fact of our pure spatial intuition from the fact of our mathematical knowledge. The latter, by contrast, establishes the fact of our pure spatial intuition independently and then derives mathematical knowledge therefrom. The latter argument is thus the stronger argument if the goal is to defend a foundational role for pure spatial intuition in mathematical reasoning.

[29] This part of the "Transcendental Exposition" is directed at our intuition of *empirical* spatial objects, and so is not included in what has traditionally been interpreted as Kant's "argument from geometry," which has always been construed to depend on his view of the necessary truth of *pure* Euclidean geometry. But, it is nevertheless important to address this passage for the sake of the current reinterpretation, since it is here that Kant makes his next philosophical move and actually begins to address the *ideality* of space.

[30] For example, the spatial orientation of a rectangle for which one side is specified as the base corresponds to the orientation between me and the surface of the table top at which I work.

The pure science of geometry thus originates in my *a priori* representation of space but nevertheless applies to spatial objects themselves (or, at least, to my *a posteriori* representation thereof). Kant's question at the start of this passage indicates that such application seems paradoxical: the successful application of pure geometry to empirical objects requires that one have an intuition of the spatial relations among outer objects prior to intuiting any particular outer object. Seeking to explain how one manages to represent the properties of things that are mind-independent prior to coming into empirical contact with those very things, he asks how an *a priori* outer intuition could be possible.

Notice that Kant is not questioning *whether* pure geometry can be so applied; as before, mathematical practice dictates his belief that pure geometry ultimately provides a structural description of certain features of empirical objects. Rather, he seeks an epistemological explanation for *how* such an application is possible. His answer requires his first articulation of what I have above identified as claim [2]: Space is a pure form of sensible intuition. That is, Kant argues that in order to explain the application of pure geometry without paradox, one must take the concept of space to be subjective, such that it has its source in our cognitive constitution. His argument here is therefore that the cognitions of geometry, which he takes to include the applicability of pure geometry to empirical objects, "are only possible under the presupposition of a given way of explaining the concept" of space, namely that our pure intuition of space is likewise the form of our outer sense. He here amplifies his claim that space is represented as a pure intuition by showing that that very representation provides us with a way to structure empirical intuitions.

As before, Kant means to suggest that a particular feature of the concept of space—that it is the form of outer sense—is able to account for the features of geometric cognition. In the first part of the "Transcendental Exposition," he showed that space as pure intuition (claim [1]) accounts for the synthetic *a priority* of geometric cognition; in this second part, he shows that space as form of sensible intuition (claim [2]) accounts for the applicability of geometric cognition. Notice then that in both parts Kant argues *from* the concept of space *to* an explanation of geometry, and not vice versa. Notice too that if the pure intuition of space that affords cognition of the principles of geometry were not also the form of our outer, sensible intuition, then the principles of geometry would have no role as a science of spatial *objects.*

The "Transcendental Exposition of the Concept of Space" and its account of our cognition of geometry has provided Kant with a bridge from the arguments of the "Metaphysical Exposition" to the doctrine of transcendental idealism. The reasoning can be summarized as follows: the "Metaphysical Exposition" shows that space is a pure intuition, or that we represent space to ourselves *a priori* and non-conceptually. Then, the first stage of the "Transcendental Exposition," the so-called "argument from geometry," shows that this pure intuition of space explains and at least partially accounts for the synthetic *a priority* of our geometric cognition.[31] Finally, the second stage of the "Transcendental Exposition" shows

[31] The claim that our pure intuition of space is a pre-condition on our cognition of geometry is insufficient, on its own, to fully account for our geometric cognition. This claim must be supple-

that to account for the *applicability* of our geometric cognition, we must see that our pure intuition of space is that "immediate representation" that allows us to form our intuitions of outer objects.

Thus, the "argument from geometry" is meant to show that a pure intuition of space provides an epistemic foundation for geometry as a synthetic *a priori* science.[32] The passage that follows the "argument from geometry" and concludes the "Transcendental Exposition" shows that the pure intuition of space that enables our geometric cognition is likewise the form of sensible intuition. It is only upon concluding the "Transcendental Exposition" that Kant explicitly introduces transcendental idealism, claiming that space represents no property of things in themselves and, equivalently, that space is nothing more than a subjective representation. So, while the "Metaphysical Exposition" gives us space as pure intuition (claim [1]) and the "Transcendental Exposition" gives us space as form of outer sense (claim [2]), Kant deploys further arguments to conclude that space is *nothing more than* a pure intuition and so *nothing more than* a form of outer sense (claim [3]). These are not arguments that I can take up here; for our purposes it is sufficient to have established that the "Transcendental Exposition," and its included "argument from geometry," is meant to show neither that space is a pure intuition, nor that space is *only* a pure intuition. The "Transcendental Exposition" provides two related explanations for the fact that we have synthetic *a priori* cognition of geometry that is applicable to objects cognized *a posteriori*: 1) geometry "flows from" the pure intuition of space; and 2) the pure intuition of space provides us with a form for intuiting real spatial objects.[33]

There are two obvious questions raised by my interpretation. First, if Kant's purpose in the "argument from geometry" is to defend neither the results of the "Metaphysical Exposition" nor the doctrine of transcendental idealism, then what motivates him to provide an account of geometric cognition at this point in the *Critique*? My primary response to this question is that, as stated above, the "argument from geometry" provides a philosophical bridge from the "Metaphysical Exposition" to transcendental idealism by moving synthetically from our *a priori* representation of space through our *a priori* knowledge of the science of space to the empirical reality and transcendental ideality of space. More generally, I see Kant's entire critical project as informed by a conception of mathematics and a

mented by an account of the sufficient conditions for the resulting cognition of geometry. That is, there remains more to say about the actual procedures that lead to geometric cognition, such as the identification of the first principles of geometry, the constructibility of geometric objects, and the deduction of geometric theorems. I will say more about this in section 3 below.

[32] I am here disagreeing with Michael Friedman's recent view on this matter: his claim that "the spatial intuition grounding the axioms of geometry is fundamentally kinematical" leads him ultimately to conclude that Kant's theory of pure spatial intuition "does not provide, *and does not attempt to provide*, an independent epistemological foundation [for geometry]" (Michael Friedman, "Geometry, Construction and Intuition in Kant and his Successors," in Gila Sher and Richard Tieszen, eds., *Between Logic and Intuition: Essays in Honor of Charles Parsons* [Cambridge: Cambridge University Press, 2000], 193). My emphasis.

[33] Were we to discard Kant's eighteenth-century conception of geometry as about the "form and extension" of *material*, spatial objects we might also choose to discard his account of the applicability of pure geometric reasoning. Nevertheless, his theory of our representation of space and related theory of the pure science of that space still stands.

corresponding philosophy of mathematics that sees mathematics as able to serve as an ideal of pure cognition. That the "argument from geometry" serves to connect the theory of space with the doctrine of transcendental idealism is thus, on my view, consistent with Kant's critical methodology.

The second question we might ask is: On what substantive basis does Kant claim that geometric cognition relies on a pure intuition of space? More specifically, does Kant have any compelling mathematical evidence that cognizing the objects of geometry requires a pure intuition of space? An answer to this question will further explain why Kant invoked geometry to play such an important role in the arguments of the "Aesthetic." It is to that question that I now turn.[34]

3. EUCLID'S GEOMETRY AS AN *A PRIORI* SCIENCE OF SPACE

Kant's reflections on mathematical practice led him to develop a compelling philosophy of geometry, which, as we have seen, played a significant role in his larger philosophical system. In this section, I will account for Kant's seemingly idiosyncratic view that geometry is grounded in a pure intuition of space by reconstructing his own reflections on the mathematical method. To do this, I will interpret Kant's observations on geometric demonstration by examining the sort of demonstrations that he himself observed, those conducted by his own mathematical contemporaries.[35] My aims in this section are modest: I will present an interesting example of Kant's philosophy of geometry in practice without here attempting to provide a complete account of that philosophy. This exercise will show more generally that the abstract connections Kant forged between his theory of space and his theory of geometry in the "Transcendental Exposition" are the basis for an interesting and coherent theory of the epistemology of spatial objects.[36]

Kant relied heavily on textbooks by Christian Wolff to learn and teach mathematics; Wolff's influence on Kant's mathematical development is patent, and it is fruitful to examine Wolff's particular mathematical practices when attempting to understand specific issues in Kant's philosophy of mathematics.[37] Wolff provides the following definition in his *Mathematisches Lexicon*: "geometry is a science of the space taken up by bodily [corporeal] things in their length, breadth, and width."[38] This definition makes it clear that the eighteenth-century geometer's objects of investigation are the shapes and sizes of ordinary objects. Accordingly, recall Kant's

[34] A third and potentially even more important question that arises is the one I hope to answer elsewhere: What in addition to a pure intuition of space is required to fully account for our geometric cognition? That is, what must we do with our representation of space to generate the axioms of geometry? See n.31 above.

[35] I have argued elsewhere (Shabel 1998; Shabel 2003) that our most fruitful strategy for understanding Kant's philosophy of mathematics and the role of mathematics in the critical philosophy requires recognition of his engagement with eighteenth-century mathematical practice.

[36] I hope to provide a full account of this theory elsewhere by exploring what I am tempted below to call the "mereotopology" provided by Euclid's geometry: the part/whole relations among spatial regions that are systematized in the definitions, postulates and common notions of the *Elements*. I hope ultimately to provide an account of the role that the Kantian notions of figurative synthesis, productive imagination, and schematism play in cognizing such relations.

[37] I explain and defend my use of, in particular, the textbooks by Christian Wolff in part 2 of Shabel 2003.

[38] Christian Wolff, *Mathematisches Lexicon* (Hildesheim: Georg Olms Verlag, 1965), 665.

comment that extension and form are two features of ordinary objects to which we have a certain privileged access: he claims that we can cognize extension and form as general, structural features of material bodies prior to cognizing those bodies empirically. This pre-empirical cognition reveals the features of the space taken up by those bodies; geometry is the result of codifying this cognition as a science.

In his own remarks on geometry, Kant regularly cites Euclid's angle-sum theorem as a paradigm example of a synthetic *a priori* judgement derived via the constructive procedure that he takes to be unique to mathematical reasoning. Euclid's theorem (*Elements* I.32) states that the interior angles of a triangle are equal to two right angles;[39] as we will see below, the demonstration is guided by constructions on a given triangle. In describing the source of the geometer's success in using intuitive reasoning to demonstrate universal and apodictic theorems such as the angle-sum theorem, Kant says:

> [as geometer I determine] my object in accordance with the conditions of either empirical or pure intuition. The former would yield only an empirical proposition (through measurement of its angles), which would contain no universality, let alone necessity, and propositions of this sort are not under discussion here. The second procedure, however, is that of mathematical and here indeed of geometrical construction, by means of which I put together in a pure intuition, just as in an empirical one, the manifold that belongs to the schema of a triangle in general and thus to its concept, through which general synthetic propositions must be constructed. (A718/B746)

Later, Kant continues by further clarifying what is involved in the geometer's intuitive reasoning, which includes the construction or exhibition of mathematical concepts in pure intuition:

> I can go from the concept to the pure or empirical intuition corresponding to it in order to assess it *in concreto* and cognize *a priori* or *a posteriori* what pertains to its object. The former is rational and mathematical cognition through the construction of the concept, the latter merely empirical (mechanical) cognition, which can never yield necessary and apodictic propositions. (A721/B749)

Notice that in both passages, Kant contrasts the results of reasoning in pure intuition as versus empirical intuition. He concludes that properly mathematical reasoning—reasoning that leads to general, synthetic, necessary, and apodictic propositions—must rely on pure intuition.

In order to interpret these passages, it will be useful to examine examples of both sorts of reasoning, and the claims that result. Kant would have been familiar with two different demonstrations of the angle-sum theorem, both presented in Wolff's texts, which illustrate the contrast between reasoning in pure versus empirical intuition. In fact, these demonstrations likewise illustrate Kant's own distinction between rational or mathematical cognition and empirical or mechanical cognition. I shall begin with what Wolff himself calls the "mechanical demonstration" of the angle-sum theorem and proceed to the properly mathematical demonstration, an analog of Euclid's own demonstration.[40]

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[39] "In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles" (Euclid, *The Thirteen Books of Euclid's Elements*, T.L. Heath, ed. and trans. [New York: Dover Publications, 1956], 316–17).

[40] Wolff's distinction between "mathematical" and "mechanical" demonstrations roughly parallels Newton's usage of these terms; see the Preface to the first edition of his *Principia* (Sir Isaac New-

In a mechanical demonstration, Wolff says, one "investigates by means of necessary instruments that which is to be proved and finds it correct."[41] Employing this technique to investigate the angle-sum theorem, Wolff first constructs a triangle ABC and adds a half-circle with center at C (Figure 1). He then "describes" the arcs *a* and *b* "with the same opening:" that is, keeping the compass open to the same degree that it was when the half-circle was described at C, Wolff constructs arcs in the interior of the triangle at angles BAC and ABC, *a* and *b* respectively. Next, Wolff "carries" the arcs *a* and *b* into the arc labeled *de*: this amounts to transferring, or re-constructing, the angles BAC and ABC in the interior of angle ACD.[42] First, angle BAC is re-constructed on AC forming ACE, then angle ABC is re-constructed on the new line CE. Wolff says that one finds, upon performing these constructions, that the two angles BAC and ABC together equal the external angle ACD; that is, the second side of the second angle will be seen to coincide with CD.[43] Consequently, the three interior angles of the triangle "fill a half circle and are equal to two right angles."

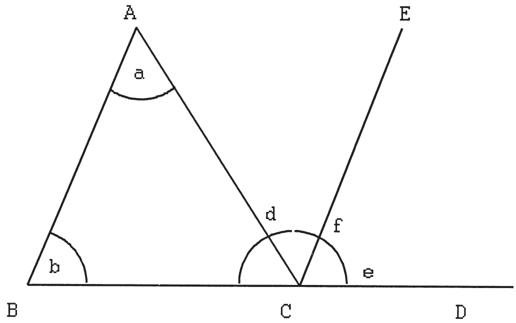

Figure 1

Kant refers directly to this demonstration in his remarks on geometry, cited above. In Kantian terms, then, the triangle, lines and arcs used to facilitate the mechanical reasoning are determined "in accordance with the conditions of empirical intuition" and "yield only an empirical proposition (through measurement of [the triangle's] angles)." The resulting propositions are neither universal nor necessary and cannot be cognized *a priori*. Thus, if there is a sense in which the angle-sum theorem can be said to have been *demonstrated* by the above argument, it is not demonstrated mathematically and so enables no properly mathematical cognition. Indeed, in the second remark quoted above, Kant follows Wolff by explicitly labeling the merely empirical cognition that results from such reasoning "mechanical."

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ton, *Principia vol. I The Motion of Bodies*, Florian Cajori, ed. [Berkeley: University of California Press, 1962], xvii). Neither Wolff's nor Newton's distinction mirrors Descartes's distinction between mathematical and mechanical curves, as stated in Book II of his *Geometrie*. An illuminating discussion of the relation between Newtonian and Cartesian conceptions of the geometrical (mathematical) and the mechanical is provided by Mary Domski in a manuscript entitled "The Constructible and the Intelligible in Newton's Philosophy of Geometry."

[41] Wolff 1965, 506.

[42] For an analysis of Wolff's procedure for "carrying" an arc, see Shabel 2003, 98ff.

[43] Note that the mechanical demonstration succeeds even if angle ABC is carried first and angle BAC second. In this case, the figure differs from that shown above, but the reasoning is identical.

Notice that in this mechanical demonstration, Wolff effectively measures the interior angles of the triangle using fallible tools: carrying the arcs via mechanical means allows the geometer to compare the size of the exterior angle to the sum of the sizes of the two opposite interior angles in a fashion that is analogous to measuring them with a protractor. This comparison is effected using what Wolff calls "Augen-Maß" (eye-measure);[44] the geometer inspects the construction in order to gather data regarding the equalities of spatial magnitudes. Thus, the judgement that the second side of the second reconstructed angle "coincides" with the extended base of the triangle, CD, is an empirical assessment based on the features of the particular constructed triangle; the skill of the geometer who "carries" the arcs; and the precision of the tools used to do so. Consequently, the assessment that the interior angles of this triangle sum to two right angles is a metric judgement based on inexact methods of demonstration, and so yields only an approximate measure of the interior angles of a particular empirically constructed object. Consequently, whatever cognition this procedure affords is neither universal nor *a priori*.

By contrast, Kant describes the sort of procedure that leads the geometer to *a priori* cognition of the necessary and universal truth of the angle-sum theorem: the object of the theorem—the constructed triangle—is in this case "determined in accordance with the conditions of . . . pure intuition."[45] The triangle is then "assessed in concreto"[46] in pure intuition and the resulting cognition is pure and *a priori*, thus rational and properly mathematical. To illustrate, I turn to Euclid's demonstration of the angle-sum theorem,[47] a paradigm case of what Kant considered *a priori* reasoning based on the ostensive but pure construction of mathematical concepts.

Euclid reasons as follows: given a triangle ABC (Figure 2), extend the base BC to D. Then construct a line through C to E such that CE is parallel to AB. Since AB is parallel to CE and AC is a transversal, angle 1 is equal to angle 1'. Likewise, since BD is a transversal, angle 2 is equal to angle 2'. It follows that the interior angles of triangle ABC are equal to the three angles that lie on the straight line BCD, or two right angles.

The reasoning here depends on three steps, not each of which is explicit: first, the geometer constructs a triangle and auxiliary lines based on definitions of primary geometric notions and previously constructed concepts; second, the geometer identifies part/whole relations among the spatial regions that have been described for the purposes of the demonstration; third, the geometer infers equality

[44] Christian Wolff, *Anfangs-Gründe aller Mathematischen Wissenschaften* (Hildesheim: Georg Olms Verlag, 1973), 161.

[45] Kant says elsewhere that "I put together in a pure intuition, just as in an empirical one, the manifold that belongs to the schema of a triangle in general and thus to its concept, through which general synthetic propositions must be constructed" (A718/B746). The details of how such a "putting together" is effected in accordance with the conditions of pure intuition are precisely what I hope to explore in the second part of this project.

[46] Note that the empirically constructed object of the mechanical demonstration is likewise "assessed *in concreto*" but the resulting cognition is *a posteriori*. The fact that the triangles in each demonstration are "concrete" objects will be addressed below.

[47] Wolff's "mathematical" demonstration of the angle-sum theorem is not relevantly different from Euclid's own.

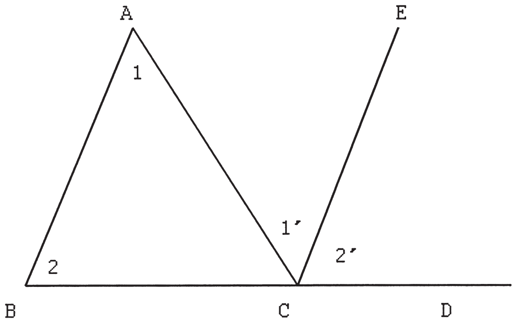

Figure 2

and inequality relations among the spatial magnitudes that are thereby represented. In particular, the base of the given triangle is extended (in accordance with postulate 2) and CE is constructed as a parallel to AB (in accordance with the previously proved proposition I.31). Upon construction of the parallel CE, the angles ACE and ECD "arise;"[48] it is a consequence of this simple construction, then, that angles ACE and ECD are parts which additively exhaust the whole exterior angle ACD. By construction, then, angles ACE and ECD together equal angle ACD. Moreover, since CD extends BC, angles BCA, ACE and ECD all lie on a single straight line. By the properties of parallel lines (proposition I.29), it thus follows that the interior angles of the triangle are equal to two right angles.

In this demonstration, as in any other properly geometrical demonstration, "Augen-Maß" is not employed. On the contrary, any predication of equalities among spatial magnitudes proceeds on the basis of a prior predication of containments among spatial regions. Further, any predication of containments among spatial regions proceeds on the basis of prior stipulations for constructing spatial regions. Finally, any construction of a spatial region (in this case a triangle, its interior and exterior angles) depends on the geometer's original ability to describe Euclidean spaces. My claim is that for Kant *this* description, ultimately, is available to us only via a pure intuition of space, that is, only via a singular, unique and immediate representation of infinite (Euclidean) space.

For Kant, the axioms or principles that ground the constructions of Euclidean geometry comprise the features of space that are cognitively accessible to us immediately and uniquely, and which precede the actual practice of geometry. These are not formal axioms, a la Hilbert, but rather principles such as those cited by Kant in the "Metaphysical Exposition": e.g., space is an infinite given magnitude; space is three dimensional; two straight lines cannot enclose a space; a triangle cannot be constructed except on the condition that any two of its sides are together longer than the third. This Kantian account of informal but contentful axioms of Euclidean geometry stemming directly from an *a priori* representation of space is itself consistent with Euclidean practice: neither Euclid's elements nor its eighteenth-century analogs offer formal axioms but rather definitions and postulates which, if taken seriously, provide a *mereotopological* description of the

[48] This is Kant's own way of putting the point. He likewise says that the geometer "obtains" these angles (A716/B744).

relations among the parts of the euclidean plane. The content of these relations is, I claim, precisely what Kant alleges is accessible to us in pure intuition, prior to geometric demonstration.[49]

In order to complete the contrast between the two methods of demonstrating the angle-sum theorem, we must make clear the role that is played by pure intuition in a mathematical context. First, the mathematical demonstration is conducted with respect to an object—a triangular space and an immediately adjacent space—the construction of which is underwritten by pre-geometric principles given by a pure intuition of space itself. Further, the steps of the mathematical demonstration include only judgements made on the basis of what is, in Kant's terms, an absolutely pure construction; thus the conclusion of the demonstration is cognized by us *a priori*. The mechanical demonstration, by contrast, is conducted with respect to an object that is empirically rendered and inspected; thus, reasoning on its basis leads us to a merely *a posteriori* conclusion. Moreover, *the mechanical reasoning is itself underwritten by the mathematical*: in the same sense in which an intuition of an ordinary object of experience is, for Kant, dependent on a prior pure intuition of space and time, so is an intuition of an empirically drawn triangle dependent on a prior pure intuition of euclidean space. Just as, for Kant, it is impossible for me to intuit an ordinary object of experience without antecedent (or at least simultaneous) awareness of its spatio-temporal form, so is it impossible for me to empirically render a particular triangular space without antecedent (or at least simultaneous) awareness of euclidean space itself.

This last observation should help to resolve what some readers might find paradoxical about the above comparison: the figures or "concrete intuitions" constructed for the two demonstrations are very nearly identical despite the fact that, on Kant's view, one is empirical and the other pure. How can they "look" the same? More importantly, what sense does it make to speak of the "look" of an *absolutely pure* intuition? We can solve this puzzle by clarifying Kant's view of the relevant contrast. The mechanical demonstration is accompanied by an actual triangular object, here a particular drawing on paper of a triangle with a particular shape and size; this concrete and particular individual object plays a direct *empirical* role in the reasoning that ensues. Indeed, one reason why the demonstration fails to produce an *a priori* defensible claim about all triangles is that its reasoning directly depends on a particular given empirical object.

By contrast, the particular empirical features of the figure that accompanies the mathematical demonstration do not themselves play any role in the reasoning; this figure is, on Kant's view, a diagram of a mental act of construction and is rendered on paper for merely heuristic reasons.[50] The drawing that accompanies Euclid's proof represents or diagrams the cognitive procedure for describing the spatial region that is delineated by a three-sided closed plane figure.[51] Kant takes

[49] See nn. 4 and 26, above.

[50] See Immanuel Kant, "On a Discovery According to which Any New Critique of Pure Reason Has Been Made Superfluous by an Earlier One" in *The Kant-Eberhard Controversy*, Henry Allison, ed. (Baltimore: The Johns Hopkins University Press, 1973), 127; Immanuel Kant, *Philosophical Correspondence 1759–99*, Arnulf Zweig, ed. (Chicago: The University of Chicago Press, 1967), 149 and 155.

[51] This is what Kant takes to be accomplished when the pure intuition "puts together . . . the manifold that belongs to the schema of a triangle in general and thus to its concept" (A718/B746).

the procedure of describing this space to be pure, or *a priori*, since it is performed by means of a prior pure intuition of space itself; my cognition of individual spatial regions is *a priori* since they are cognized *in*, or as limitations on, the essentially single and all-encompassing space itself.

4. CONCLUSION

For Kant, pure intuition of space enables pure intuition of geometric objects. Recalling the eighteenth-century understanding of geometry as the science of spatial magnitude, it follows that, for Kant, pure intuition of space enables pure intuition of spatial magnitudes. Insofar as the spatial magnitudes described by geometry represent the spatial features of three-dimensional objects (thereby individuating objects in space), it follows further that pure intuition of space enables intuition of at least some features of the spatial objects of our ordinary experience.

On standard readings of the "argument from geometry," we fail to make a coherent connection between Kant's theory of space as pure intuition and his theory of geometrical reasoning. This is primarily because, on standard readings, we suppose that Kant meant to *defend* his theory of space on the basis that it provides the best explanation for our knowledge of geometry. I have argued instead that Kant intended his so-called "argument from geometry" not as a *defense* of his theory of space, but rather as an independent account of the dependence of geometric reasoning on a pure intuition of space. On Kant's view, our ability to construct geometric objects and investigate their properties depends on our having a pure intuition of space. In the "Transcendental Exposition," however, Kant is not using this dependence to argue *for* our having a pure intuition of space; rather, he is offering an account of geometric knowledge that forges a connection between a prior theory of space as pure intuition and a future theory of space as form of sensible intuition. I am suggesting, then, that we not read Kant's argument in the "Transcendental Exposition" as primarily a transcendental argument toward the conclusion that space is a pure intuition, but rather as an explanation of the role of space as pure intuition in our practice of geometry. If this explanation provides further evidence that we have a pure intuition of space, then so much the better for Kant's theory of space. But I hope I have shown that his theory of space does not stand or fall with the "argument from geometry" and, moreover, that the "argument from geometry" provides a coherent and compelling account of the role that space as pure intuition plays in justifying geometric knowledge.

On my reading, Kant would deny that we use geometric reasoning to access our pure intuition of space, in favor of affirming that we use our pure intuition of space to attain geometric knowledge. To make this point more generally, I should emphasize that, for Kant, pure spatial intuition provides an epistemic starting point for the practice of geometry. Since geometry is, for Kant and his contemporaries, the foundational mathematical discipline,[52] pure spatial intuition thus con-

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Notice that this procedure does afford us some knowledge of triangles, namely that they have an interior and an exterior, three angles, and three sides. But notice also that this procedure does not afford us any knowledge of the particular features of the triangle that represents the mental act: we can predicate no particular measures of the sides or angles.

[52] I provide an argument for this in part 2 of Shabel 2003.

stitutes an epistemological foundation for the mathematical disciplines as Kant understood them. This epistemological foundationalism amounts to a view according to which our *a priori* mental representation of space provides us with the original cognitive data for our geometric investigations, investigations which ultimately produce a mathematical theory of the empirical world.

The following important issue remains unresolved: what larger philosophical purpose can Kant's account of geometric cognition serve? Recall that the "Transcendental Exposition" includes the claim that the cognitions that "flow from" the concept of space, namely the cognitions of geometry, "are only possible under the presupposition of a given way of explaining the concept" of space. We have seen above what such a presupposition amounts to: *a priori* cognition of geometry as Kant understood it depends on space being both a pure intuition and also the form of outer sense. This claim forces Kant to assess the extent of the knowledge of outer objects provided by the science of geometry. Thus he is led directly to the arguments in the "Conclusions from the Above Concepts" where he attempts to show that space is *nothing more than* the form of outer sense (claim [3]). While a discussion of these arguments for a full-fledged transcendental idealism must be postponed, I do not believe that the "argument from geometry" will be implicated should we choose to reject their conclusions. That is, we can accept Kant's theory that our representation of space is absolutely *a priori* and non-conceptual, as well as his explanation of the role that this representation plays in the eighteenth-century science of Euclidean geometry, without concluding that space itself is transcendentally ideal.[53]

So, whether or not the philosophical bridge Kant has built leads him to a defensible doctrine of transcendental idealism, I hope to have shown nonetheless that the bridge itself supports a rich and important theory of the relation between our representation of space and our cognition of Euclidean geometry. Kant argues that geometric cognition relies essentially on principles that are accessible to us via a pure intuition of space. Cognition of these principles grounds the practice of Euclidean geometry; moreover, cognition of Euclidean geometry grounds a theory of the cognition of ordinary spatial objects. If we take this seriously as a philosophy of the sort of mathematics with which Kant was engaged, as I think we must, then our next step must be to evaluate the broad and deep role that this ideal of mathematical cognition plays in the remainder of Kant's critical philosophy.[54]

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[53] Careful assessment of this conclusion would necessarily include reflection on the relation between the perceptually accessible "space" that is occupied by ordinary objects of our experience and the more universal "space" that is posited by astrophysicists. I suspect that, given Kant's epistemological goals, we will be forced, at the very least, to limit Kant's conclusion to the former. Notice that this limitation will also help us to retain the "argument from geometry" in spite of notorious developments in non-Euclidean geometry.

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